رویکردی جامع برای ارزیابی اقتصادی در پروژه
همراه با آموزش مدلسازی در اکسل (Excel)

بخش سیزدهم:

تحلیل حساسیت، سر به سر، عدم قطعیت و ریسک: تصمیم گیری تحت ریسک

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What we will learn?

- Expected value analysis
  - Estimate probability and expected value
    - Mean
    - Variance
    - Standard deviation
    - If scenario
    - Example 1
  - Example 2
  - Example 3
  - Example 4
  - Example 5

What we will learn?

- Staged evaluation of alternatives using a decision tree
  - Example 6
  - Real options in engineering economics
    - ✓ Example 7
    - ✓ An option
    - ✓ A real option
    - ✓ Real options analysis
    - ✓ Industrial setting examples
    - ✓ Personal decision making examples
    - ✓ Primary characteristics
  - Example 8

Decision making under risk

- Now the **element of chance** is formally **taken into account**. However, it is more **difficult to make a clear decision** because the analysis attempts to accommodate **variation.**

- **Fundamentally**, there are **two ways** to consider **risk in an analysis**:

1. **Expected value analysis**
2. **Simulation analysis**

Expected value analysis

- **Engineers** and **economic analysts** usually deal with

  1. **Estimate variation**
  2. **Risk**

  about an **uncertain future** by **placing appropriate reliance** on **past data**, if any exist.

This means that ***probability*** and ***samples*** are used.

Expected value analysis

- Actually the use of **probabilistic analysis** is **not as common** as might be **expected**.
  - The reason is **not that the computations are difficult** to perform or understand, but that **realistic probabilities** associated with **cash flow estimates** are **difficult to assign**.

- **Experience** and **judgment** can often be used in conjunction with **probabilities and expected values** to evaluate the desirability of an alternative.

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Expected value analysis

- Use the **<u>chance</u>** and **<u>parameter</u> estimates** to calculate **expected values *E* (parameter)** via formulas.
- Analysis results in ***E* (cash flow)**, ***E* (AOC)**; and the final result is the expected value for a measure of worth, such as ***E* (PW), *E* (AW), *E* (ROR), *E* (BC).** To select the alternative, **choose the most favorable expected value** of the measure of worth.

The **expected value** can be interpreted as a long-run average observable if the project is repeated many times. Since a particular alternative is evaluated or implemented only once, the expected value results in a **point estimate.** However, even for a single occurrence, the expected value is a meaningful number.

The expected value $E(X)$ is computed using the relation

$$E(X) = \sum_{i=1}^{i=m} X_i P(X_i)$$

where X_i = value of the variable X for i from 1 to m different values

$P(X_i)$ = probability that a specific value of X will occur

Simulation analysis

- Use the **chance** and **parameter** estimates to **generate repeated computations** of the **measure of worth** relation **by randomly sampling** from a **plot for each varying parameter**.

- When a **representative** and **random sample** is complete, **an alternative** is selected **utilizing a table** or **plot of the results**.

- Usually, **graphics** are an **important part** of decision making via **simulation analysis**.

Estimate probability and the expected value

- A random variable is a function which assigns a value to each event included in the set of all possible events.

$$P(A) \geq 0$$

$$P(A) = 1 \left(\text{Certain Event} \right)$$

$$P(A.B) = P(A) \times P(B) \left(\text{Independent Events} \right)$$

$$P(A+B) = P(A) + P(B) \left(\text{Mutually Exclusive Events} \right)$$

$$P(A/B) = P(A.B) / P(B) \left(\text{Conditional Probability} \right)$$

Estimate probability and the expected value

- Heads: H
- Tails: T
  - Two tosses of a coin:

$$P(H) = 50\%$$

$$P(T) = 50\%$$

$$P(H1, H2) = 0.50 \times 0.50 = 0.25$$

$$P(H1, T2) = 0.50 \times 0.50 = 0.25$$

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$$P(T1, T2) = 0.50 \times 0.50 = 0.25$$

Estimate probability and the expected value

- Heads: H
- Tails: T
  - **Two heads:** P(H1, H2) = 0.50 × 0.50 = 0.25

$P(H) = 50\%$

$P(T) = 50\%$

$P(H1, H2) = 0.50 \times 0.50 = 0.25$

$P(H1, T2) = 0.50 \times 0.50 = 0.25$

$P(T1, H2) = 0.50 \times 0.50 = 0.25$

$P(T1, T2) = 0.50 \times 0.50 = 0.25$

Estimate probability and the expected value

- Heads: H
- Tails: T
  - **One head:** P(H1, T2) + P(T1, H2) = 0.25 + 0.25 = 0.50

$$P(H) = 50\%$$

$$P(T) = 50\%$$

$$P(H1, H2) = 0.50 \times 0.50 = 0.25$$

$$P(H1, T2) = 0.50 \times 0.50 = 0.25$$

$$P(T1, H2) = 0.50 \times 0.50 = 0.25$$

$$P(T1, T2) = 0.50 \times 0.50 = 0.25$$

Estimate probability and the expected value

- Heads: H
- Tails: T
  - **No head:** $P(T1, T2) = 0.50 \times 0.50 = 0.25$

$$P(H) = 50\%$$

$$P(T) = 50\%$$

$$P(H1, H2) = 0.50 \times 0.50 = 0.25$$

$$P(H1, T2) = 0.50 \times 0.50 = 0.25$$

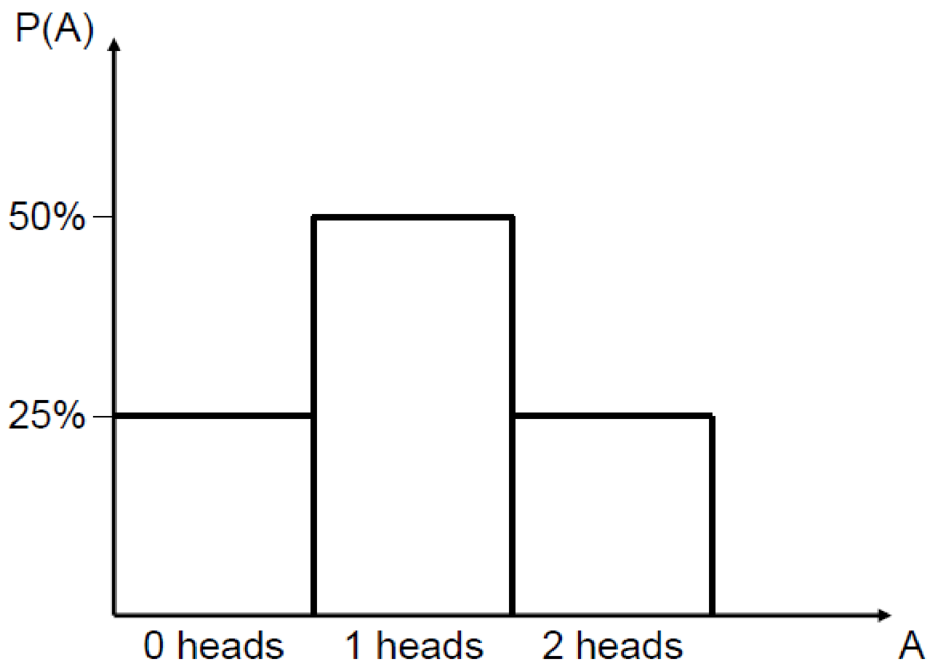
$$P(T1, H2) = 0.50 \times 0.50 = 0.25$$

$$P(T1, T2) = 0.50 \times 0.50 = 0.25$$

Estimate probability and the expected value

$0 \le P(A) \le 1$

$\Sigma P(A) = 1$



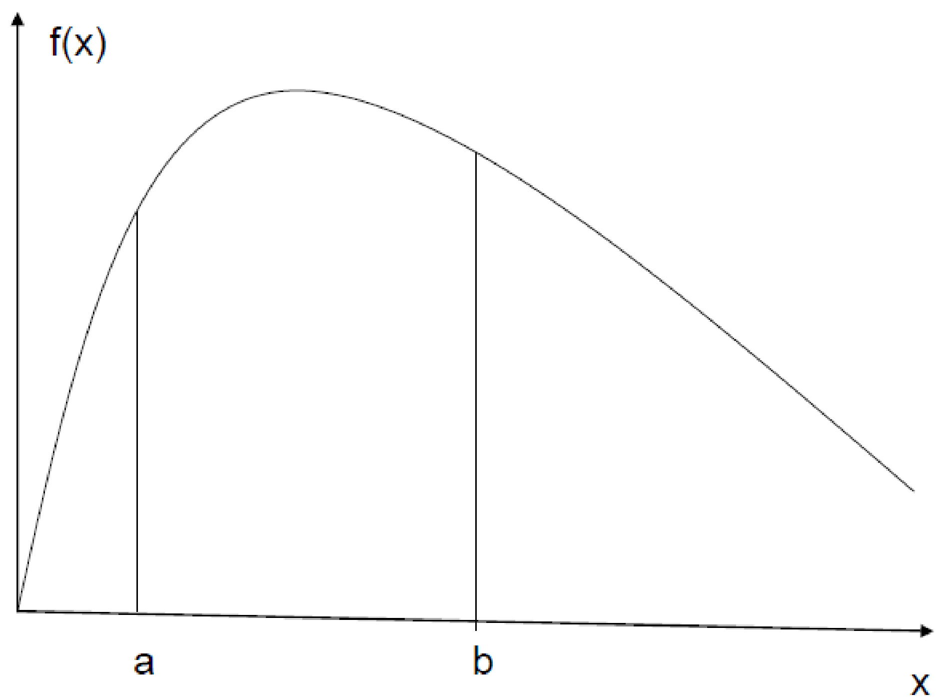
Estimate probability and the expected value

- Probability density function (continuous events)

$$0 \le f(x) \le \infty$$

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$P(a \le x \le b) = \int_{a}^{b} f(x)dx$$

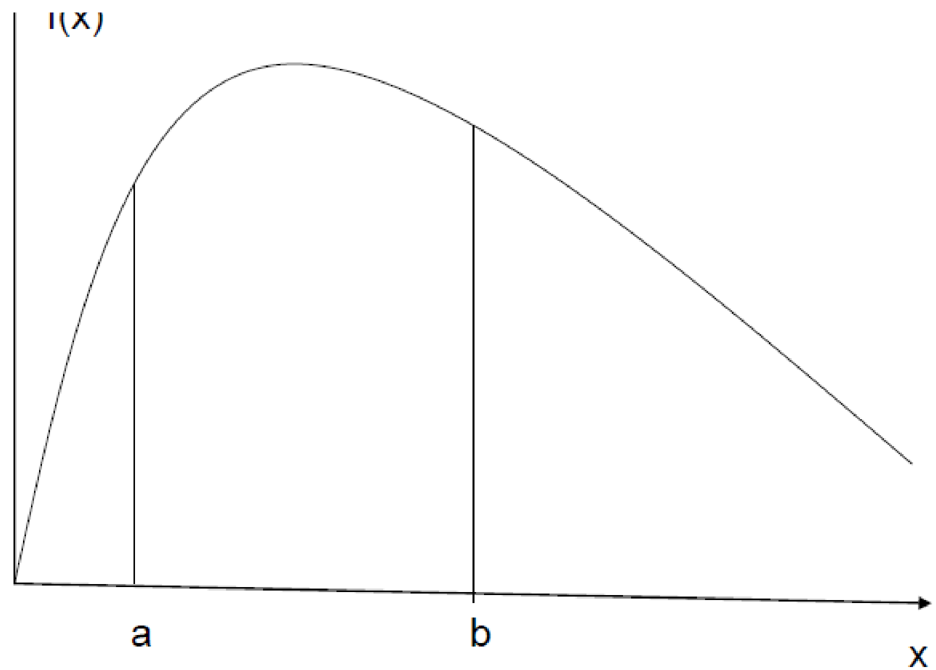


Estimate probability and the expected value – Mean (expected value)

- Indicates central tendency or concentration of mass

$$E(x) = \sum_{x} x \times P(x) \quad \text{for a discrete random variable}$$

$$E(x) = \int_{-\infty}^{\infty} x \times f(x)dx \quad \text{for a continuous random variable}$$



Estimate probability and the expected value - Variance

❑ Variance (var)

❖ It is a measure of the spread of the dispersion of the probability distribution

$$Var(x) = E(x^2) - [E(x)]^2$$

For a discrete random variable x :

$$Var(x) = \sum_x x^2 \times P(x) - \left[\sum_x x \times P(x)\right]^2$$

For a continuous random varaible x :

$$Var(x) = \int_{-\infty}^{\infty} x^2 \times f(x)dx - \left[\int_{-\infty}^{\infty} x \times f(x)dx\right]^2$$

$$Var = (\sum (AW, PW, FW)_i^2 \times P_i) - Expectedvalue^2)$$

Estimate probability and the expected value – Standard deviation

- Standard deviation (SD or σ)
  - It is the square root of the variance

$$\sigma = \sqrt{Var(x)}$$

  - In statistical analysis:
    - Sum numbers -- average -- subtract the average from each number -- use exponent of 2 for each number -- sum numbers = average

Estimate probability and the expected value – If scenario

IF:

- ✓ The **variance** is **<u>very high</u>**, we are **very uncertain** about the result
- ✓ The **variance** is **<u>very low</u>**, we are **more certain** about the result

IF:

- ✓ Obtained **<u>expected value</u>** is identical for **all alternatives**, we use
  1. Alternative with **lower variance** which represents **higher certainty**
  2. Alternative with **lower probability** in making **loss**

Estimate probability and the expected value – Example 1

Example 1: Coin will be tossed twice.

- ✓ If no heads occur, $100 will be lost
- ✓ If one heads occurs, $40 will be won
- ✓ If two heads occur, $80 will be won

What is the expected value of the profit from the game using the random variable "number of heads"?

Expected value

$$E(G) = -(\$100 \times 0.25) + (\$40 \times 0.50) + (\$80 \times 0.25) = -25 + 20 + 20 = \$15$$

Variance

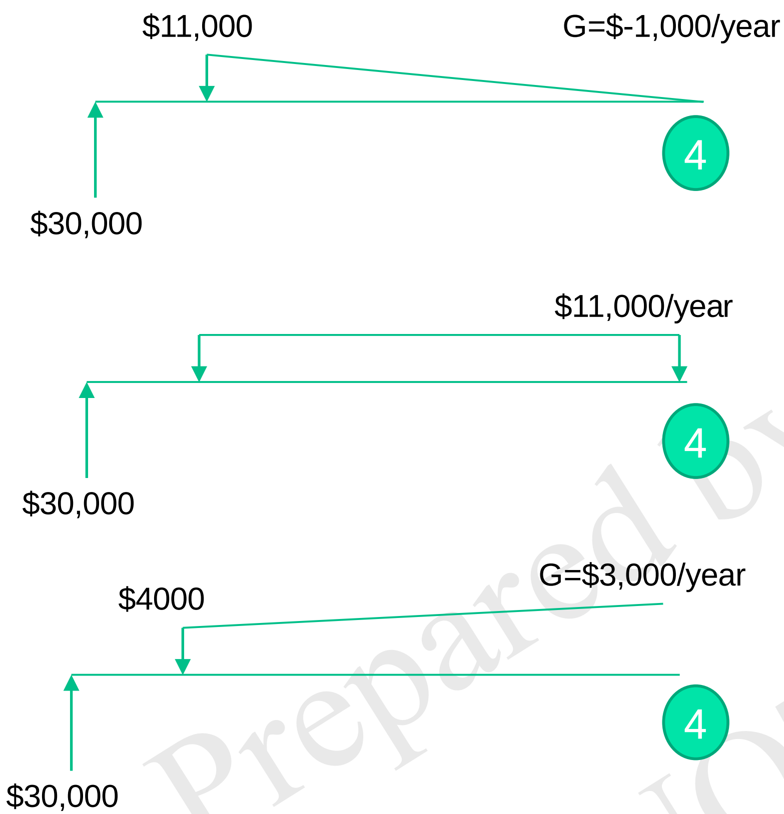
$$Var(G) = E(G^2) - [E(G)]^2 = ((-\$100)^2 \times 0.25) + (\$40^2 \times 0.50) + (\$80^2 \times 0.25) - \$15^2$$
$$= 2{,}500 + 800 + 1{,}600 - 225 = \$4{,}675$$

Decision making under risk – Example 2

Example 2: An excavation contractor wants to introduce a new machine to his fleet that is similar to an existing machine. After a detailed study of the marketing and technical departments, estimates are made of possible future cash flows as related to varying market conditions. Cash Flow (A) will occur if demand decreases, Cash Flow (B) if demand remains constant, and Cash Flow (C) if demand increases. **Based on past experience** and **projections of future** economic activity, it is believed that the **probabilities** of decreasing, constant, and increasing demand will be 0.10, 0.30, and 0.60 respectively.

In Cash Flow (A), the machine will bring revenue of $11,000 in the first year but due to lower demand in the following years, this value will decrease at a rate of $1,000 per year. In Cash Flow (B), because of constant demand, there will be constant revenue of $11,000 per year during the entire life of the machine. Finally, due to increasing demand in Cash Flow (C), the income will be $4,000 in the first year, and will increase at a rate of $3,000 per year. The life of the machine is 4 years, and its initial cost is $30,000 in all three cases.
This firm generally uses the present worth criterion to make such decisions and MARR is 10%. Develop the probability mass function. Calculate the expected value of present worth.

Decision making under risk – Example 2



Demand decrease – A – P(A) = 10%

Demand is constant - B – P(B) = 30%

Demand increase – C – P(C) = 60%

$$\sum_{k=0}^{n} = \%100$$

All conditions are mutually exclusive events

Decision making under risk – Example 2

P(A+B+C) = P(A) + P(B) + P(C)

PW_A(%10) = -30000 + 11000 (P/A,10,4) – 1000 (P/G,10,4) = $492

PW_B(%10) = -30000 + 11000 (P/A,10,4)= $4870

PW_C(%10) = -30000 + 4000(P/A,10,4) + 3000(A/G,10,4)(P/A,10,4) = - $4185

$E[PW(\%10)] = (\$492 * 0.1) + (\$4870 * 0.3) - (\$4185 * 0.6) = -\$1001 \leq 0$ not desirable

Decision making under risk – Example 3

Example 3: A company has a large water treatment facility located in the flood plain of a river. The construction of a levee to protect the facility during periods of flooding is under consideration.

Using historical records that describe the maximum height reached by the river during each of the last 50 years, the frequencies shown in Column B of the following table were determined. From these frequencies are calculated the probabilities that the river will reach a particular level in any one year. The probability for each height is determined by dividing the number of years for which each particular height was the maximum by 50, the total number years.

The damages that are expected if the river exceeds the height of the levee are related to the amount by which the river height exceeds the levee. These costs are shown in Column D. If the flood crest is 15 ft and the levee is 10 ft, the anticipated damages for a difference of 5 ft will be $100,000, whereas a flood crests of 20 ft for a levee 10 ft high (difference 10 ft) would create damages of $150,000.

The cost of constructing levees of various heights is shown in Column E. The company considers a MARR of 12% and it is felt that after 15 years the treatment plant will be relocated away from the flood plain. The company wants to select the levee height that minimizes its total expected costs.

Decision making under risk – Example 3

HEIGHT, X (ft)	No OF YEARS RIVER LEVEL WAS X ft ABOVE NORMAL	PROBABILITY OF RIVER BEING X ft ABOVE NORMAL	LOSS IF RIVER LEVEL IS X ft ABOVE LEVEE	INITIAL COST OF BUILDING LEVEE X ft HIGH
A	B	C	D	E
0	24	24/50=0.48	$0	$0
5	12	12/50=0.24	100,000	100,000
10	8	8/50=0.16	150,000	210,000
15	3	3/50=0.06	200,000	330,000
20	2	2/50=0.04	300,000	450,000
25	1	1/50=0.02	400,000	550,000
TOTAL	50 years	1.00		

Decision making under risk – Example 3

Build no levee

$\text{Annual investment cost} = \$0$

$$\text{Expected annual damage} = (0 * 0.48) + (100{,}000 * 0.24) + (150{,}000 * 0.16) + (200{,}000 * 0.06) + (300{,}000 * 0.04) + (400{,}00 * 0.02) = \frac{\$80{,}000}{\text{yr}}$$

$\text{Total expected annual cost} = \text{investment cost} + \text{expected damage} = 0 + 80000 = \&80{,}000/\text{yr}$

Build 5ft levee

$$\text{Annual investment cost} = 100{,}000\left(\frac{A}{P}, 12{,}15\right) = \$14{,}680/\text{yr}$$

$$\text{Expected annual damage} = (0 * 0.48) + (0 * 0.24) + (100{,}000 * 0.16) + (150{,}000 * 0.06) + (200{,}000 * 0.4) + (300{,}000 * 0.02) = \$39{,}000/\text{yr}$$

$\text{Total expected annual cost} = \$14{,}680 + \$39{,}000 = \$53{,}680/\text{yr}$

Decision making under risk – Example 3

Build 10ft levee

$$\text{Annual investment cost} = 210{,}000\left(\frac{A}{P}, 12{,}15\right) = \$30{,}828/\text{yr}$$

$$\text{Expected annual damage} \\ = (0 * 0.48) + (0 * 0.24) + (0 * 0.16) + (100{,}000 * 0.06) + (150{,}000 * 0.4) + \\ (200{,}000 * 0.02) = \$16{,}000/\text{yr}$$

$$\text{Total expected annual cost} = \$30{,}828 + \$16{,}000 = \$46{,}828/\text{yr}$$

Decision making under risk – Example 3

Levee Height(ft)	Annual investment cost ($/yr)	Expected annual damage (AE/yr)	Total expected annual cost
0	$0	$80,000	$80,000
5	$14,682	$39,000	$53,682
10	$30,828	$16,000	**$46,828**
15	$48,450	$7,000	$55,450
20	$66,069	$2,000	$68,069
25	$80,751	0	$80,751

Decision making under risk – Example 4

Example 4: A firm has a set of four mutually exclusive alternatives from which one is to be selected. The probability mass functions describing the likelihood of occurrence of the present worth amounts for each alternative are given below.

Which alternative should be selected? Decide by using variances.

ALTS	PW				
	-$40,000	$10,000	$60,000	$110,000	$160,000
A_1	0.2	0.2	0.2	0.2	0.2
A_2	0.1	0.2	0.4	0.2	0.1
A_3	0.0	0.4	0.3	0.2	0.1
A_4	0.2	0.2	0.3	0.2	0.1

Decision making under risk – Example 4

$$E(PW_A) = (-40{,}000 * 0.1) + (10{,}000 * 0.2) + (60{,}000 * 0.4) + (110{,}000 * 0.2) + (16{,}000 * .01)$$
$$= 60{,}000$$

Alt<	Expected PW
A_1	$60,000
A_2	$60,000
A_3	$60,000
A_4	$60,000

یا واریانس کمتر را انتخاب می کنیم یا **Alt** با احتمال ضرر کمتر

$$VAR(PW_{A2})$$
$$= ((-40{,}000)^2 * 0.1 + (10{,}000)^2 * 0.2 + (60{,}000)^2 * 0.4$$
$$+ (110{,}000)^2 * 0.2 + (160{,}000 * 0.1)) - 60{,}000^2 = 3{,}000{,}000$$

Alt<	Expected PW
A_1	$5,000,000
A_2	$3,000,000
A_3	**$2,500,000**
A_4	$32,600,000

Decision making under risk – Example 5

Example 5: A dam is being planned for a certain river of erratic flow. Five alternative designs (A, B, C, D and E) are being considered that will cost \$142,000, \$154,000, \$170,000, \$196,000 and \$220,000 to build respectively and will require annual maintenance amounting to \$4,600, \$4,900, \$5,400, \$6,500 and \$7,200 respectively. It has been determined by past experience that the probabilities of flows exceeding the dams' respective capacities are 0.10, 0.05, 0.025, 0.0125 and 0.00625 for dams A, B, C, D and E, in which cases the expected annual damage will amount to \$122,000, \$133,000, \$145,000, \$170,000 and \$190,000. The life of the dam will be 40 years with no salvage value. For an interest rate of 4%, which one of the five dam designs will result in minimum expected cost.

Decision making under risk – Example 5

Initial cost	Annual maintenance	Probabilities of damage	A=40 years Annual damage
$C_A = \$142{,}000$	$AM_A = \$4{,}600/yr$	$P(A) = \%10$	$AD_A = \$122{,}000/yr$
$C_B = \$154{,}000$	$AM_B = \$4{,}900/yr$	$P(B) = \%5$	$AD_B = \$133{,}000/yr$
$C_C = \$170{,}000$	$AM_C = \$5{,}400/yr$	$P(C) = \%2.5$	$AD_C = \$145{,}000/yr$
$C_D = \$196{,}000$	$AM_D = \$6{,}500/yr$	$P(D) = \%1.25$	$AD_D = \$170{,}000/yr$
$C_E = \$220{,}000$	$AM_E = \$7{,}200/yr$	$P(E) = \%0.625$	$AD_E = \$190{,}000/yr$

Decision making under risk – Example 5

$$\mathrm{E}[AE_A(\%4)] = \$4{,}600 + \$142{,}000\left(\frac{A}{P},4{,}40\right) + \$122{,}000 * 0.1 = \$23{,}974/yr$$

$$\mathrm{E}[AE_B(\%4)] = \$4{,}900 + \$154{,}000\left(\frac{A}{P},4{,}40\right) + \$133{,}000 * 0.05 = \$19{,}330/yr$$

$$\mathrm{E}[AE_C(\%4)] = \$17{,}613/yr$$

$$\mathrm{E}[AE_D(\%4)] = \$18{,}527/yr$$

$$\mathrm{E}[AE_E(\%4)] = \$19{,}502/yr$$

Staged evaluation of alternatives using a decision tree

- **Alternative evaluation** may require **a series of decisions** in which the **outcome from one stage** is **important to the next stage** of **decision making**.

- When **each alternative** is clearly **defined** and **probability estimates** can be made to **account for risk,** it is helpful to perform the evaluation using a **decision tree.**

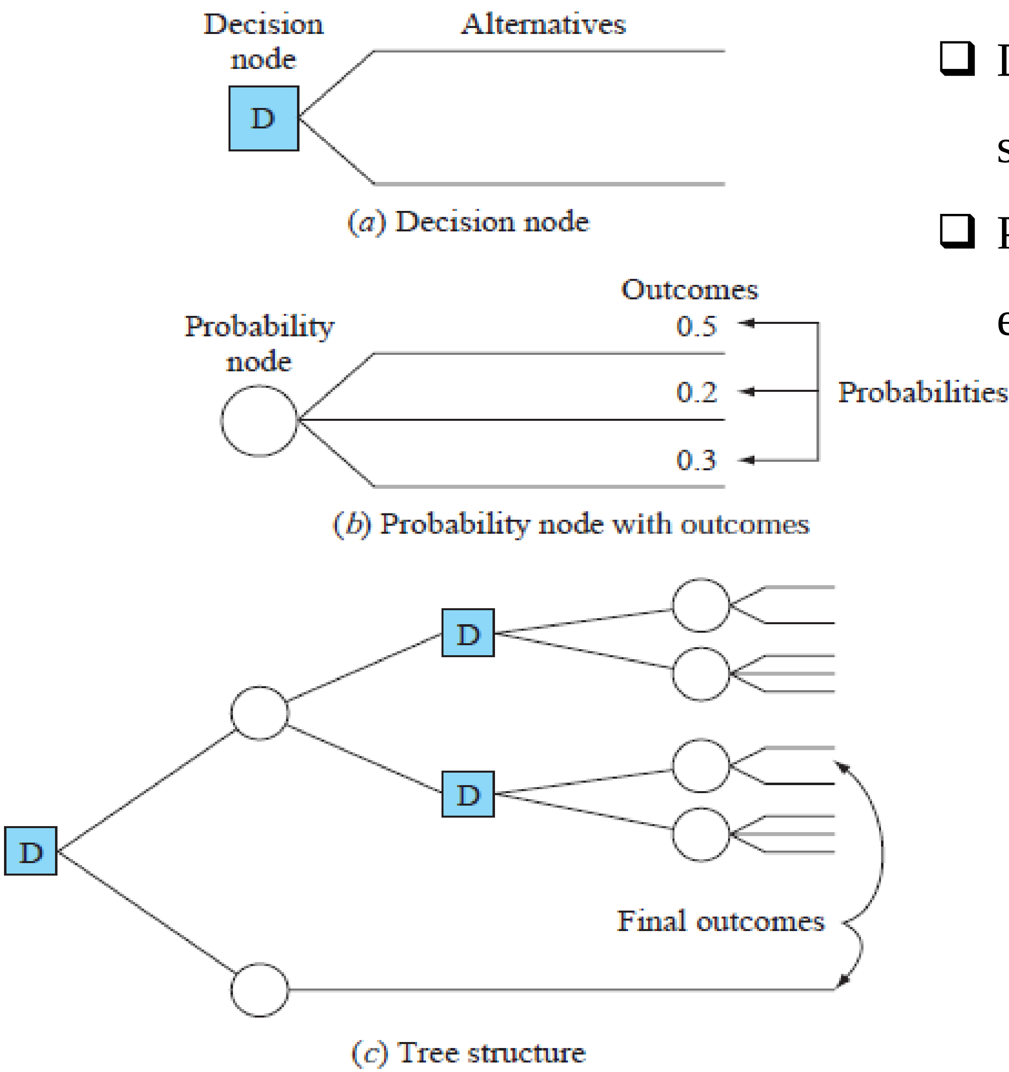
- **Decision trees** are used for **decision problems** where consideration of **sequences of decisions** is **important** and **probabilities of future events are known**.

Staged evaluation of alternatives using a decision tree

A decision tree includes:

- More than one stage of alternative selection.
- Selection of an alternative at one stage that leads to another stage.
- Expected results from a decision at each stage.
- Probability estimates for each outcome.
- Estimates of economic value (cost or revenue) for each outcome.
- Measure of worth as the selection criterion, such as E(PW).

Staged evaluation of alternatives using a decision tree



Decision and probability nodes used to construct a decision tree.

- ❑ Decision node says which alternative is selected.
- ❑ Probability node says the probability that each alternative associated with.

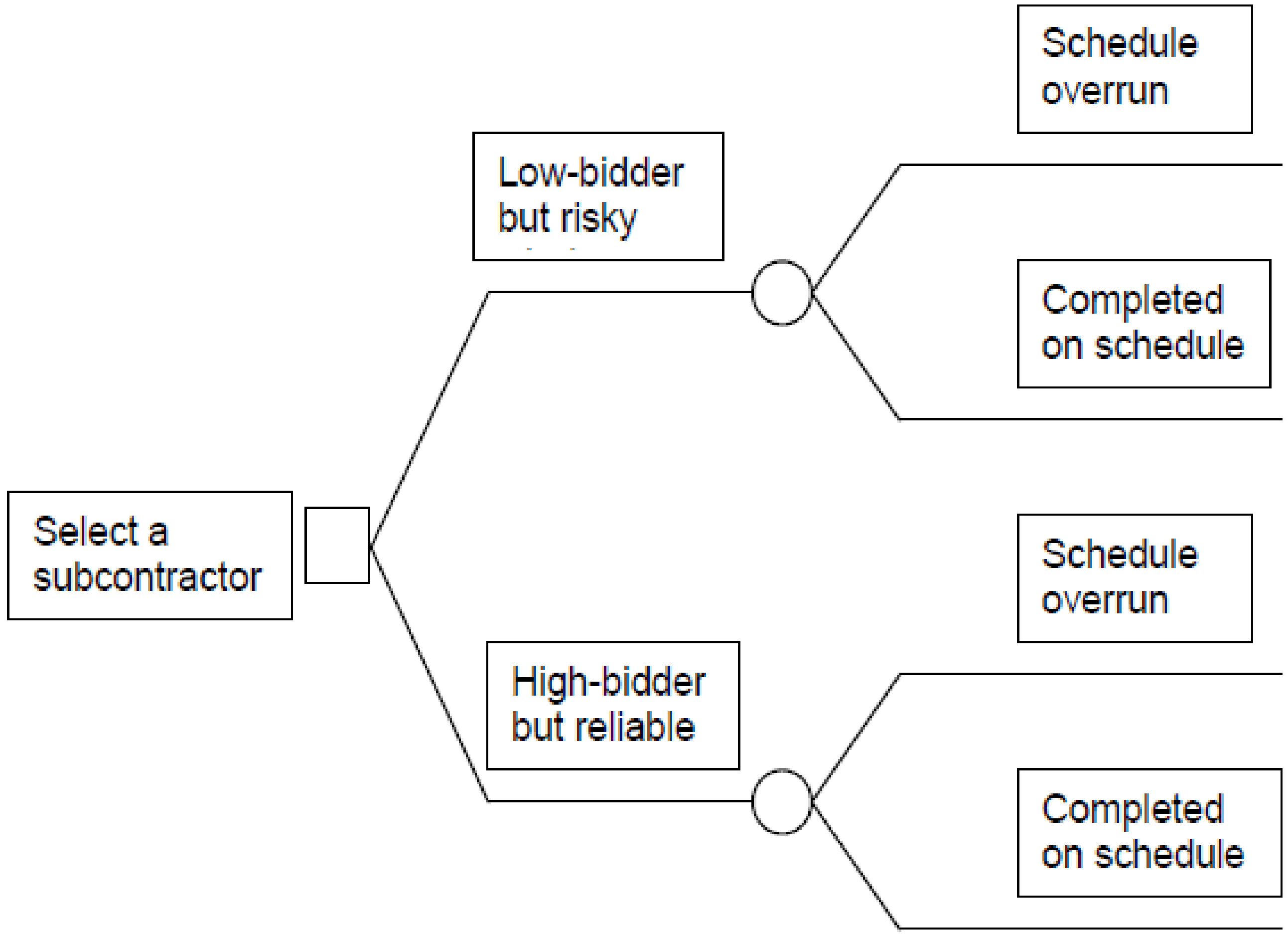
Staged evaluation of alternatives using a decision tree – Example 6

Example 6: We are the prime contractor on a project and there is a penalty in our contract with the owner that depends on how many days we are late. We need to decide which subcontractor to use for a critical activity.

- One subcontractor that we have not worked with before presents a lower cost ($120,000) bid. We estimate however that there is a 60% chance that this contractor will be late, incurring a penalty of $50,000.
- The higher-cost contractor bids $140,000. We know this contractor and assess that it poses only a 30% chance of being late, with a late penalty of $30,000.
  - Use decision tree analysis to decide which subcontractor to select?

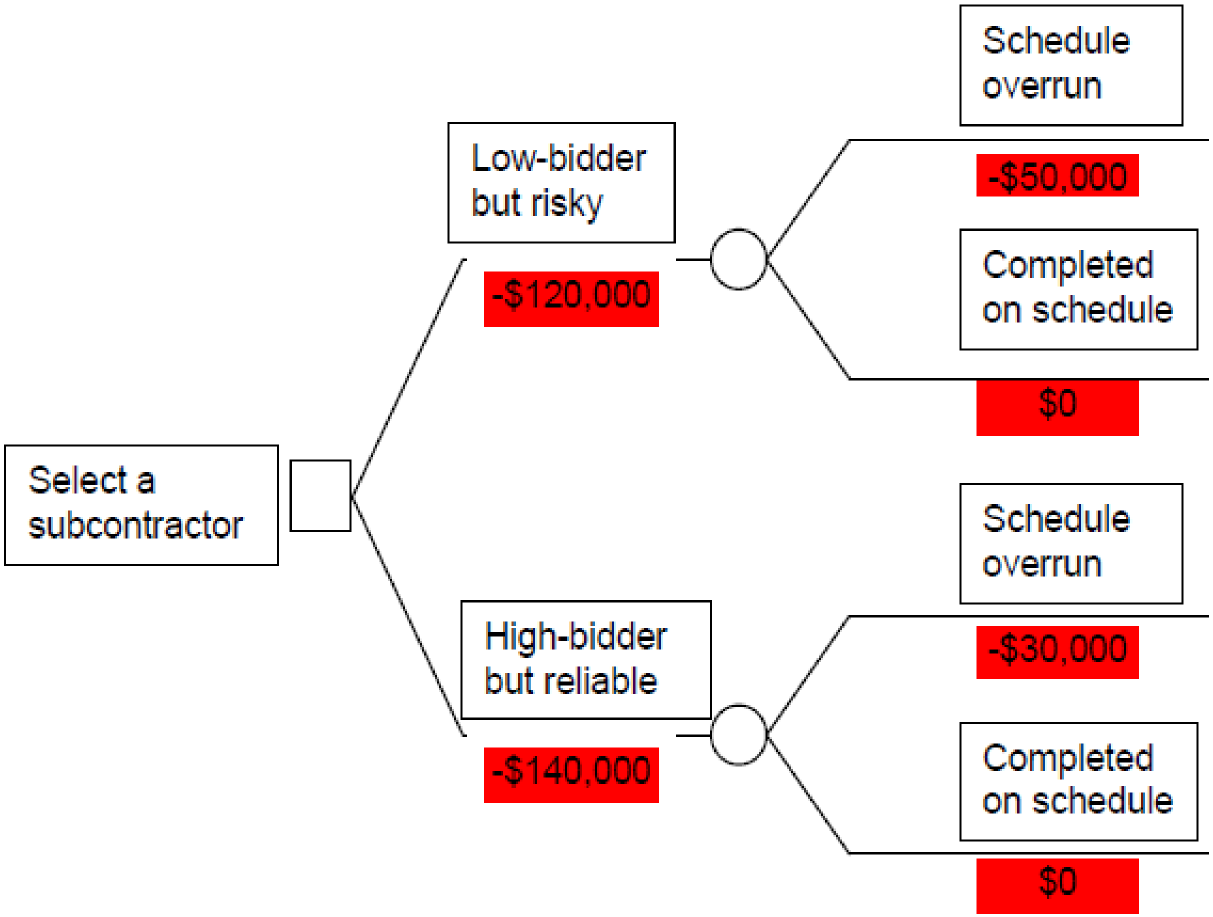
Staged evaluation of alternatives using a decision tree – Example 6

STEP 1



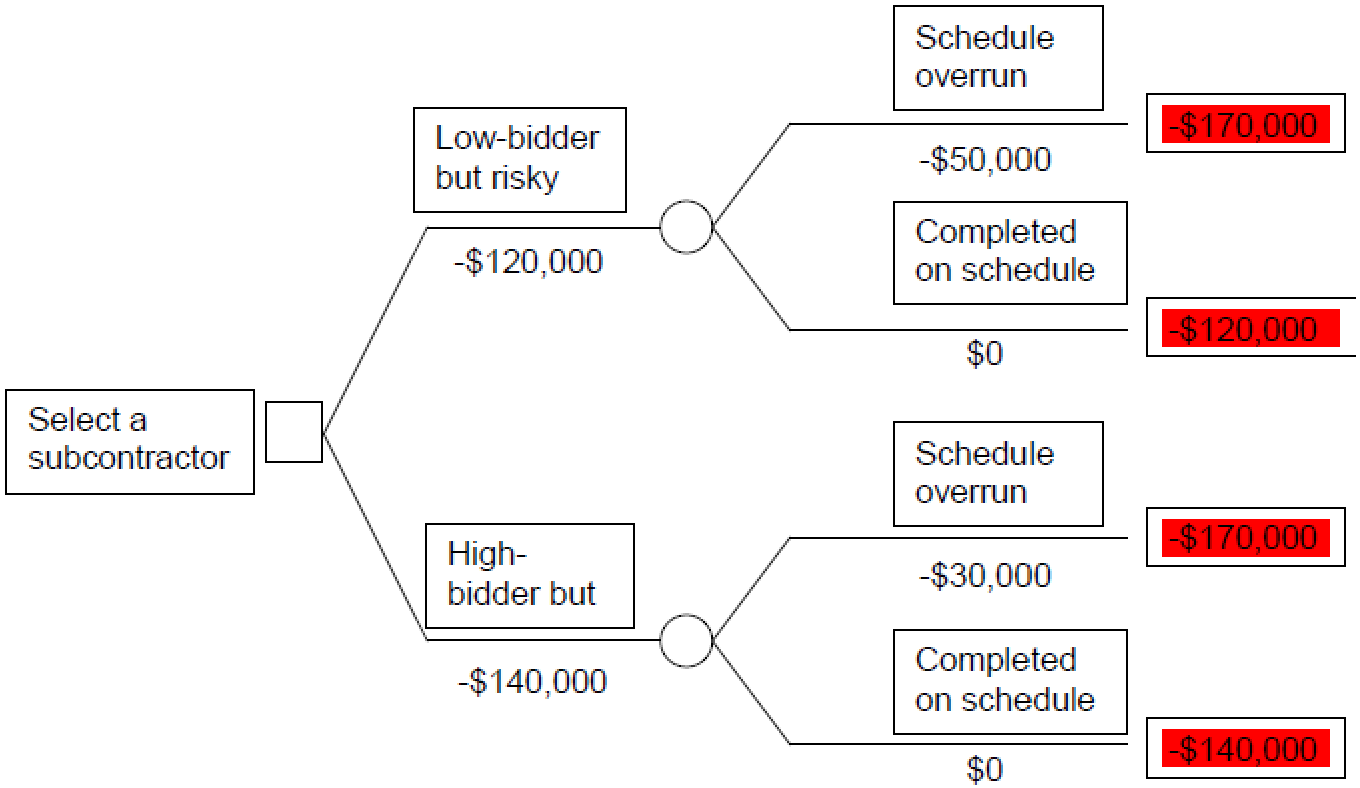
Staged evaluation of alternatives using a decision tree – Example 6

STEP 2



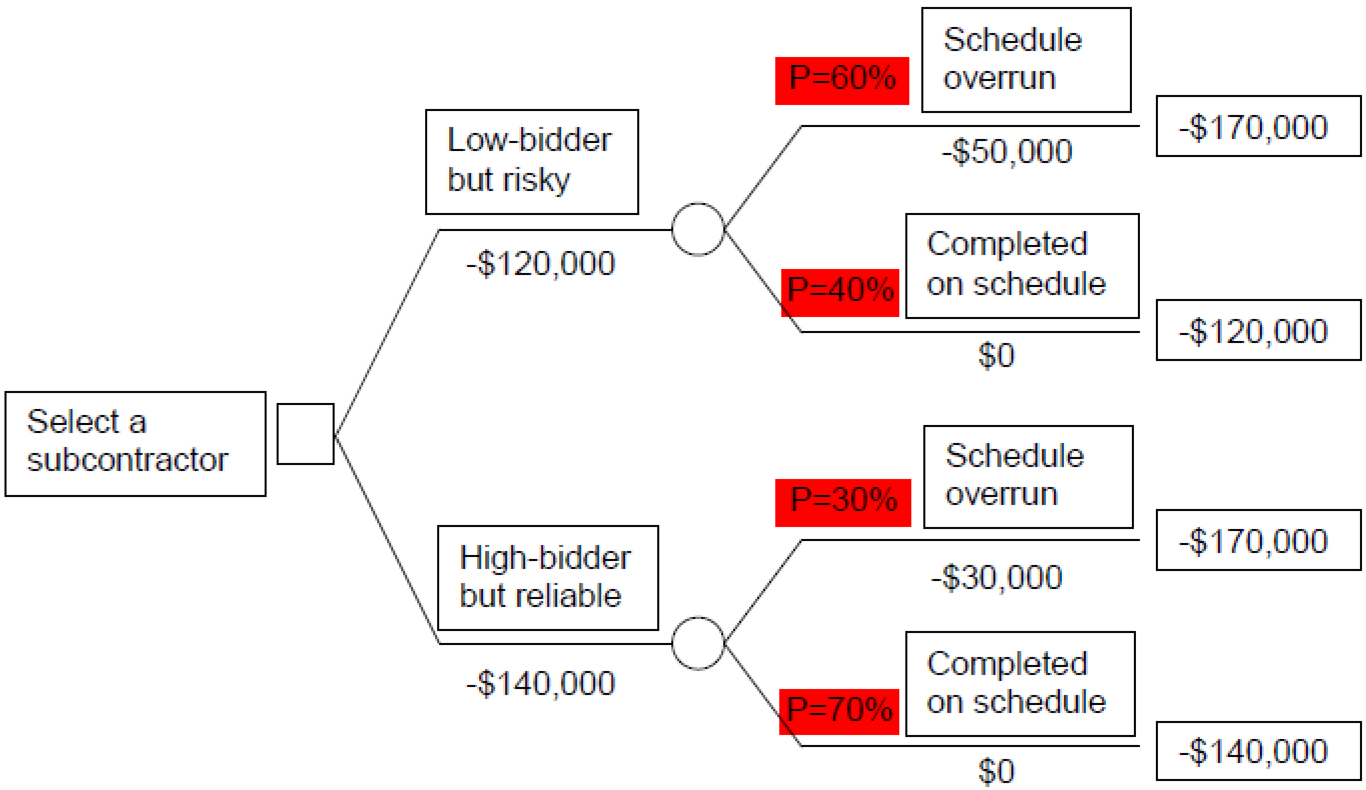
Staged evaluation of alternatives using a decision tree – Example 6

STEP 3



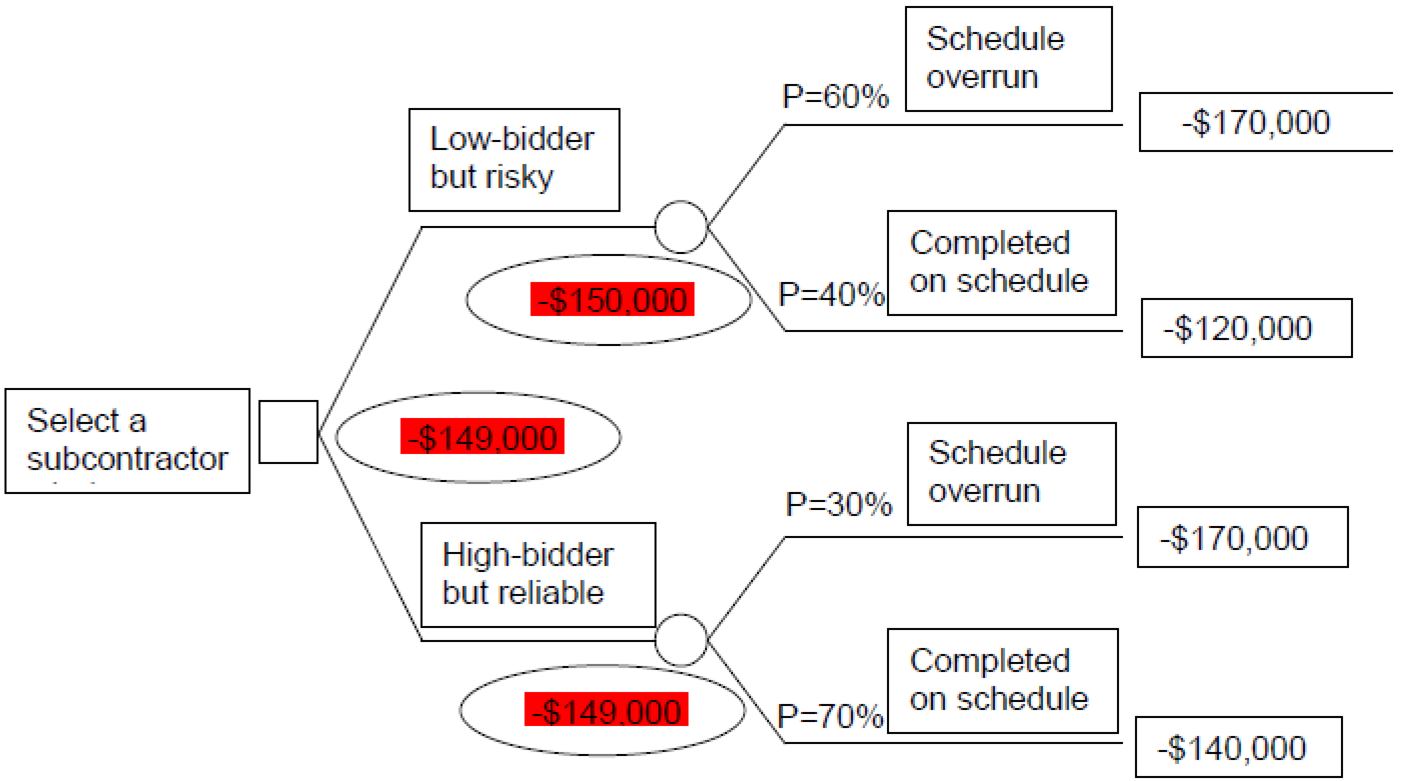
Staged evaluation of alternatives using a decision tree – Example 6

STEP 4



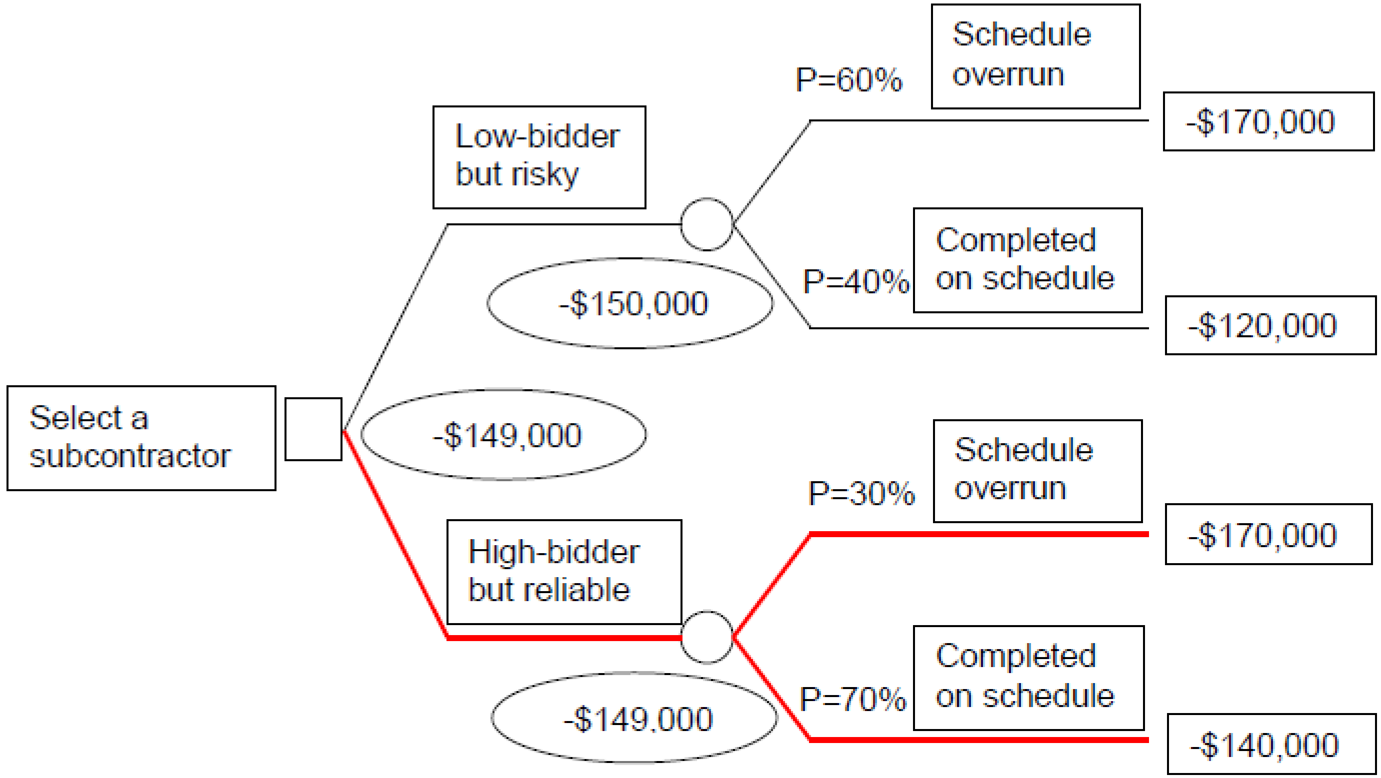
Staged evaluation of alternatives using a decision tree – Example 6

STEP 5



Staged evaluation of alternatives using a decision tree – Example 6

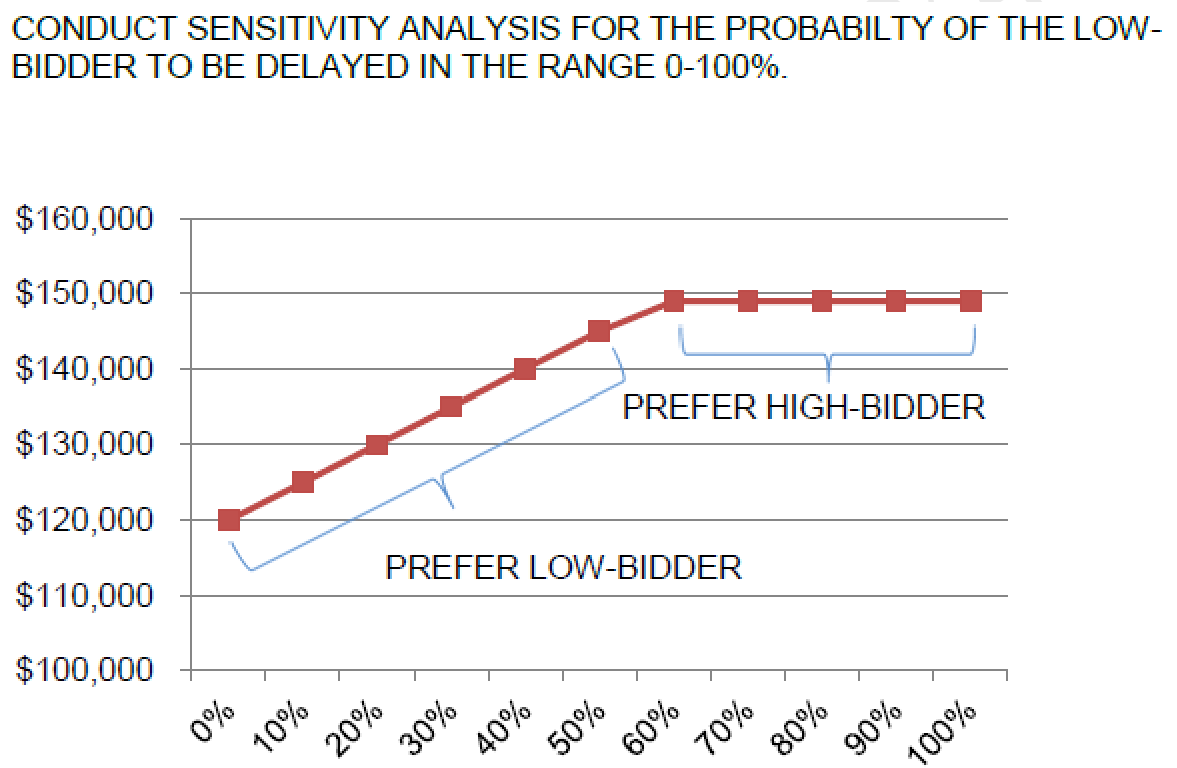
STEP 6: -$149,000 is called the expected cost.



EXPECTED VALUE OF COST AT TOP AND BOTTOM CHANCE NODES

Staged evaluation of alternatives using a decision tree – Example 6

STEP 7: what if the chance of schedule overrun becomes 50% for low bidder? Low bidder will be preferred.



Real options in engineering economics

- **Many of the problems** in engineering economy can be viewed as **staged decisions**.

- When the **decision to invest** **more or less can be delayed into the future**, the problem is called **staged funding.**

Real options in engineering economics – Example 7

Example 7: Assume a large company expects to sell an energy-saving, window-mounted residential air conditioning unit at the rate of 100,000 per month by the end of 2 years on the market. Decision makers may opt to:

(1) Build the capacity to supply100,000 per month to market immediately or

(2) Build capacity to supply 25,000 per month now and test the market's receptivity.

- If positive, they can stage the increase by 25,000 additional units each 6 months to meet current demand.
- Of course, if an aggressive competitor enters the scene, or the economy falters, the staged funding decision will change as warranted.

These alternatives provide time-based options to the company.

Before we go further, some definitions are needed.

Real options in engineering economics – An option

An **option** is a purchase or investment that contractually provides the privilege to take a stated action by some stated time in the future, or the right to not accept the offer and forfeit the option.

Real options in engineering economics – A real option

A **real option**, in engineering economy terms, is the investment (cost) in a project, process, or system. The options usually involve **physical (real)** assets, buildings, equipment, materials, and the like, thus, the word *real*. Options may also be leases, subcontracts, or franchises. The investment alternatives present varying amounts of **risk**, which is estimated by probabilities of occurrence for predictable future events.

Real options in engineering economics – Real options analysis

Real options analysis is the application of techniques to determine the economic consequences of **delaying** the funding decisions as allowed by the option. The estimated cash flows and other consequences of these delays are analyzed with risk taken into account to the degree possible. A measure of worth, e.g., PW or AW, is the criterion used to make the staged funding decisions. A decision may be to expand, continue as is, contract, abandon, or replicate the alternative at the time the option must be exercised.

Real options in engineering economics

- An <u>**inherent part of real options analysis**</u> is the **uncertainty of future estimates**, as it is for **most economic analysis**.

- After some illustrations of real options, let's discuss the **probabilistic dimensions**.

Samples from industry and everyday personal life that can be formulated as real options

Real options in engineering economics – Industrial setting

1. **New markets**—Purchase equipment to enter an expanding international market over the next 5 years.
2. **New planes**—Purchase commercial airplanes now with an option to buy an additional 5 planes over the next 3 years at the same price as that paid for the current order.
3. **Removing car models**—Ford Motor Company can decide to maintain production on an established car model with dwindling sales for the next 3 years or can opt to discontinue the model in stages over a 1- or 2-year period.
4. **Drilling lease**—Buy a drilling option contract from landowners to drill for oil and gas at some time in the next 10 years. The drilling may not be justified at this time, but the contract offers the option to drill where it to become economically advantageous based on events such as increased oil prices or improved recovery technology.

Real options in engineering economics – Personal decision making

1. **Extended car warranty**—When purchasing a new car, the option to buy an extended-coverage warranty beyond the manufacturer’s warranty is always an option. The price of the option is the cost of the extended warranty. The uncertainties and risks are the future unknown costs for repairs and failed components.

2. **House insurance**—When a homeowner has no mortgage to pay, maintaining house insurance is an option. Deductibles are high enough, e.g., 1% to 5% of the fully appraised value, that insurance primarily covers only catastrophic damage to the structure. Self-insurance, where money is set aside for potential damages while accepting the risk that a major event will take place, is an option for the homeowner.

Real options in engineering economics – Primary characteristics

- Some of the **primary characteristics (with an example)** of a **real options analysis** performed within the **context of engineering economics** are as follows:

1. **Cost to obtain the option to delay a decision**
   - PW of initial investment, lease cost, or future investment amount
2. **Anticipated future options and cash flow estimates**
   - Double production with annual net cash flow estimated
3. **Time period for follow-on decisions**
   - Staged decision time, such as 1 year or a 3-year test period

Real options in engineering economics – Primary characteristics

- Some of the **primary characteristics (with an example)** of a **real options analysis** performed within the **context of engineering economics** are as follows:

4. **Market and risk-free interest rates**
    - Expected market MARR of 12% per year and inflation rate estimate of 4% per year
5. **Estimates of risk and future uncertainty for each option**
    - Probability that an estimated cash flow series will actually occur, if a specific option is selected
6. **Economic criterion**
    - PW, ROR, or other measure of worth

Real options in engineering economics – Important note

It is common to use a **decision tree** to **record** and **understand** the **options** **prior** to performing a **real options analysis** with **risk included**.

Example 8

Example 8: A contracting company is planning to start building prefabricated housing units. Because prefabricated building construction is somewhat different than the company's existing activities, it will be necessary to construct a precast components plant to start the business.

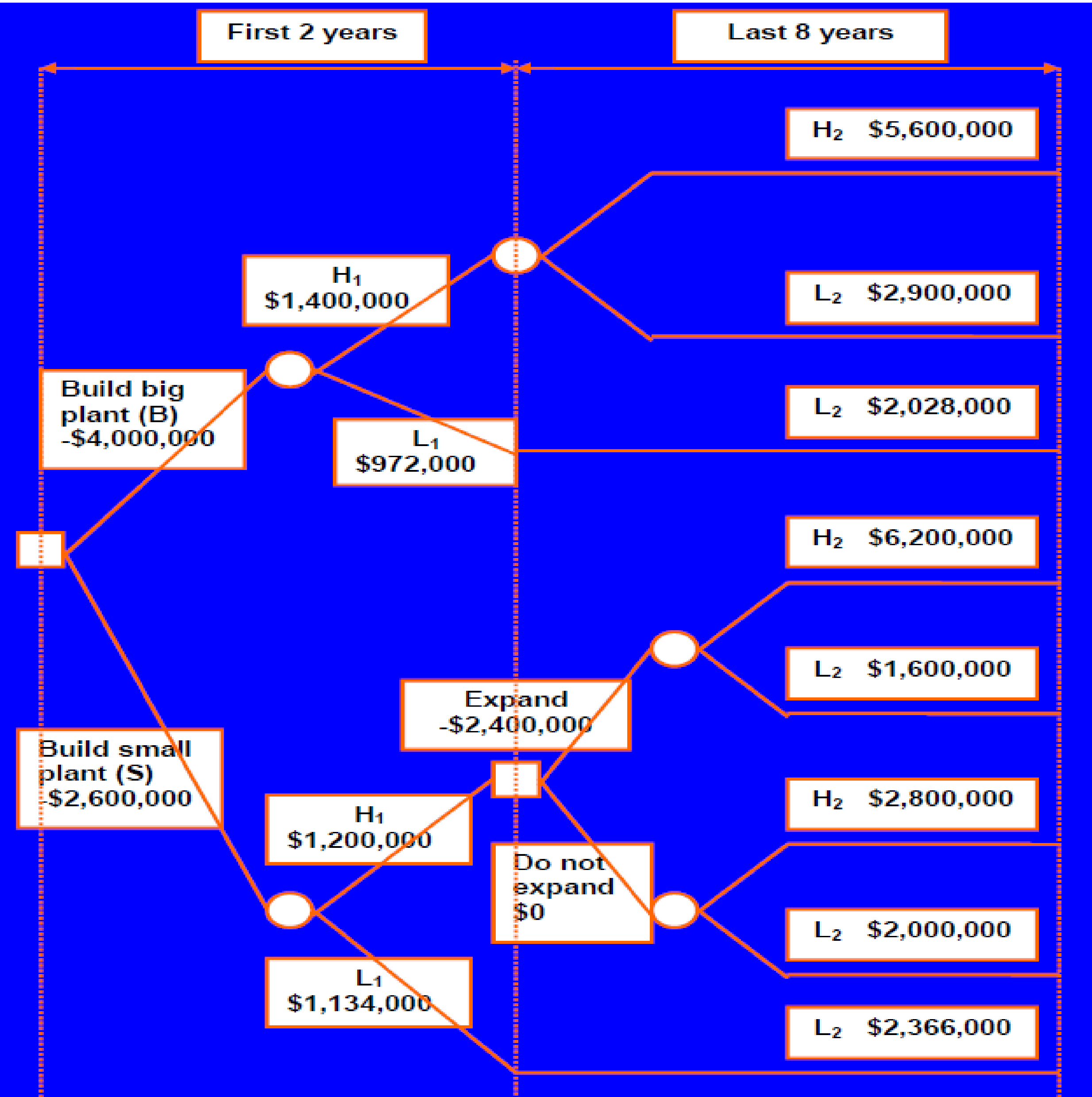
Based on the information supplied by the company's marketing group, it is believed that the demand for this type of housing may change over the next 10 years.

1. If there is a **high** demand for prefabricated housing over the **next 10 years**, the product is considered to be a **good seller**.
2. If the demand is **high in the first two** years and **low in the remaining eight years**, the product is considered to be an **average seller**.
3. If the demand is **low for the next 10 years**, the product is considered to be a **poor seller**.

Example 8

- **The costs associated with each case are as follows (in present worth values):**
- ✓ Build big plant : $4,000,000 initial cost
- ✓ Build small plant : $2,600,000 initial cost
- ✓ Expand small plant : $2,400,000 cost in present worth
- **Income in the first two years (in present worth values):**
- ✓ High demand with big plant : $1,400,000
- ✓ Low demand with big plant : $972,000
- ✓ High demand with small plant : $1,200,000
- ✓ Low demand with small plant : $1,134,000
- **Income in the remaining eight years (in present worth values):**
- ✓ High demand, big plant, following high demand : $5,600,000
- ✓ Low demand, big plant, following high demand : $2,900,000
- ✓ Low demand, big plant, following low demand : $2,028,000
- ✓ High demand, small plant, after expansion : $6,200,000
- ✓ Low demand, small plant, after expansion : $1,600,000
- ✓ High demand, small plant, without expansion : $2,800,000
- ✓ Low demand, small plant, without expansion : $2,000,000
- ✓ Low demand, small plant, following low demand : $2,366,000

Example 8



STEP1: Calculate the present worth values of all expenses and revenues by using discounted cash flow formulas.

Example 8

STEP 2: Sum up the values along each path to find the values at the tips.

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Example 8

- Therefore, the demand for this sort of housing will follow one of the following demand patterns. Also, the marketing department estimates the probabilities of prefabricated housing being a good, average, or bad seller as follows:

	DEMAND IN FIRST 2 YEARS	DEMAND IN LAST 8 YEARS	DESIGNATION	PROBABILITY OF OCCURRENCE
Good Seller	High (H_1)	High (H_2)	(H_1, H_2)	2/5 (i.e., 40%)
Average Seller	High (H_1)	Low (L_2)	(H_1, L_2)	1/5 (i.e., 20%)
Poor Seller	Low (L_1)	Low (L_2)	(L_1, L_2)	2/5 (i.e., 40%)

- The first alternative is to build a large plant that would be adequate for the full 10 years. The second alternative is to build a small plant and after two years of observing the sales, make a decision whether to expand it or not.

Which alternative to select is the decision problem confronting the management?

Reminder from probabilities

- Independent events (mutually exclusive)

P(H1) = P(H1, H2) + P(H1, L2)

P(L1) = P(L1, H2) + P(L1, L2)

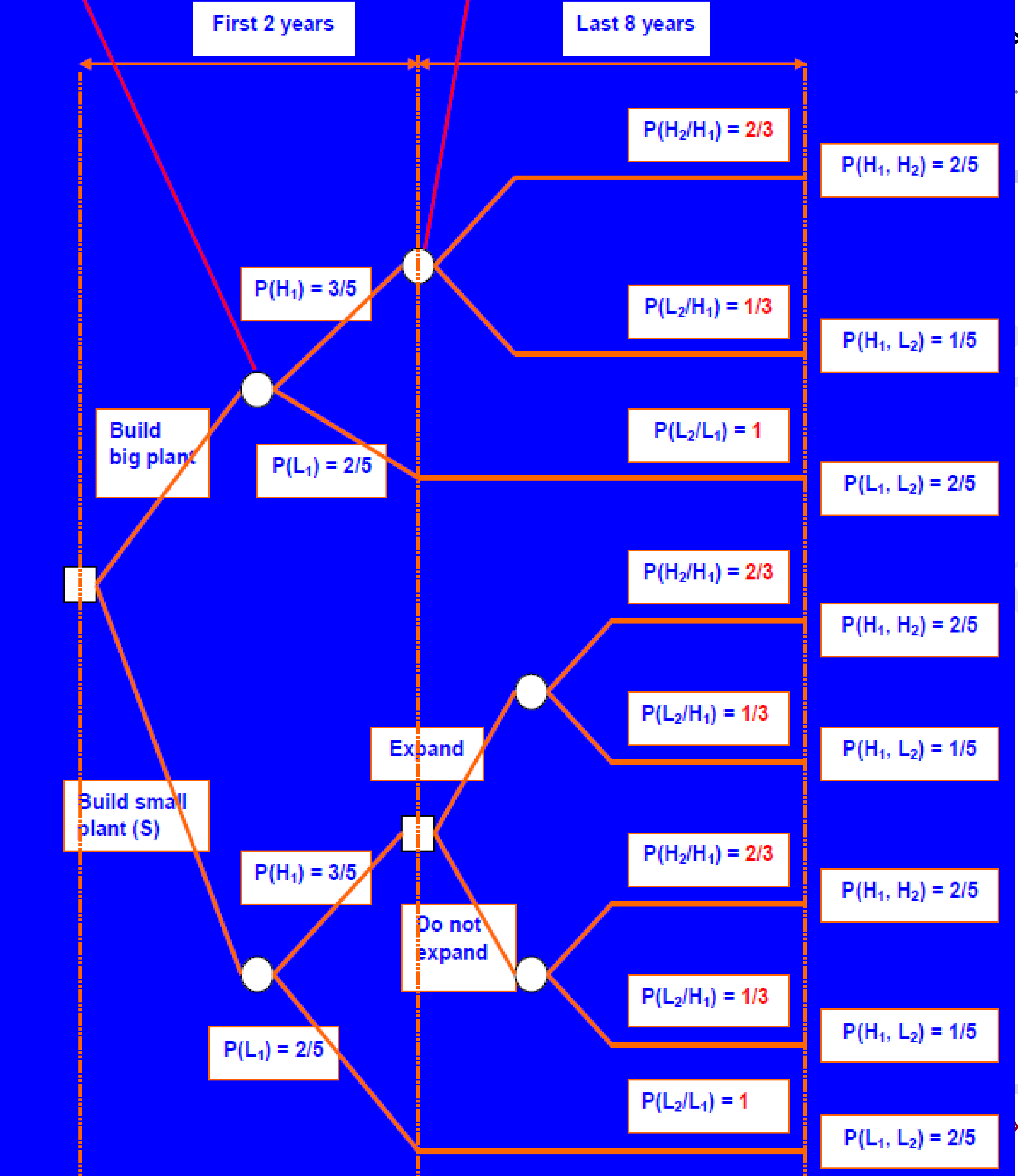
- Dependent events (conditional)

P(H2/H1) = P(H1, H2) / P(H1)

P(L2/H1) = P(H1, L2) / P(H1)

P(H2/L1) = P(L1, H2) / P(L1)

P(L2/L1) = P(L1, L2) / P(L1)



Example 8

STEP 3: Calculate the probabilities associated with each branch of the tree.

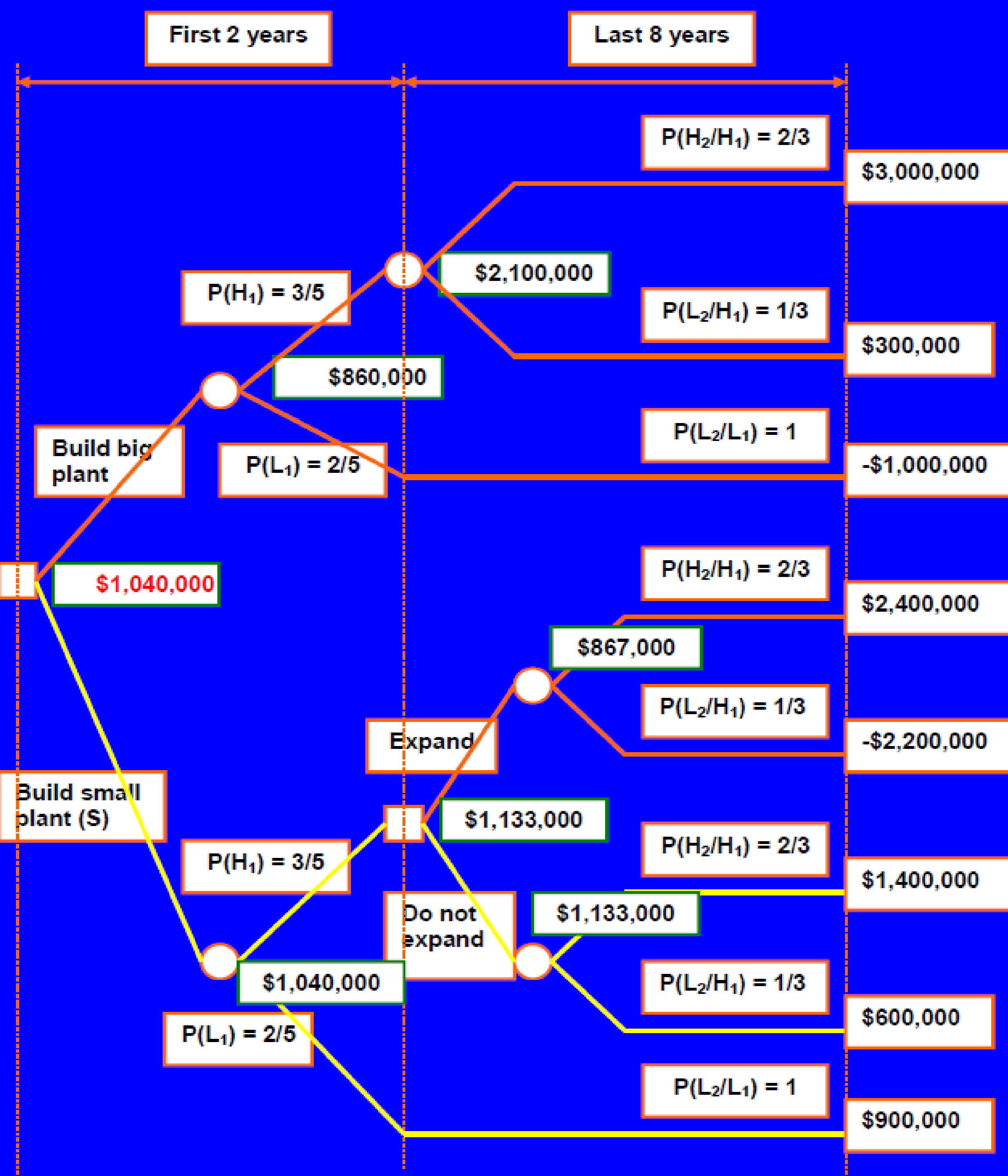
P (H1) = P(H1, H2) + P(H1, L2)
= 2/5 + 1/5
= 3/5

P (L1) = P(L1, H2) + P(L1, L2)
= 0 + 2/5
= 2/5

P (H2/H1) = (2/5) / (3/5) = 2/3

- The sum on each node should be equal to 1.

Example 8



STEP 4: Identify the decision path that leads to the most profitable solution.

- ✓ Build the small plant
- ✓ If demand is high in the first two years, the plant should not be expanded.
- ✓ If demand is low in the first two years, the plant should not be expanded.
- ✓ If this policy is followed, the expected profit will be $1,040,000.

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