رویکردی جامع برای ارزیابی اقتصادی در پروژه
همراه با آموزش مدلسازی در اکسل (Excel)

بخش سوم:

ارزیابی اقتصادی: کلیات مفاهیم ارزش زمانی پول

ارائه دهندگان:

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What we will learn?

- Time value of money
- How lenders and borrowers make decision
- Measure of worth
- Interest rate
- Terminology and symbols
- Cash flow
- Simple interest
- Compound interest
- Nominal and effective interest rates
- How to solve problems using EXCEL

Compound interest – Arithmetic gradient factors

$\frac{Find}{Given}$

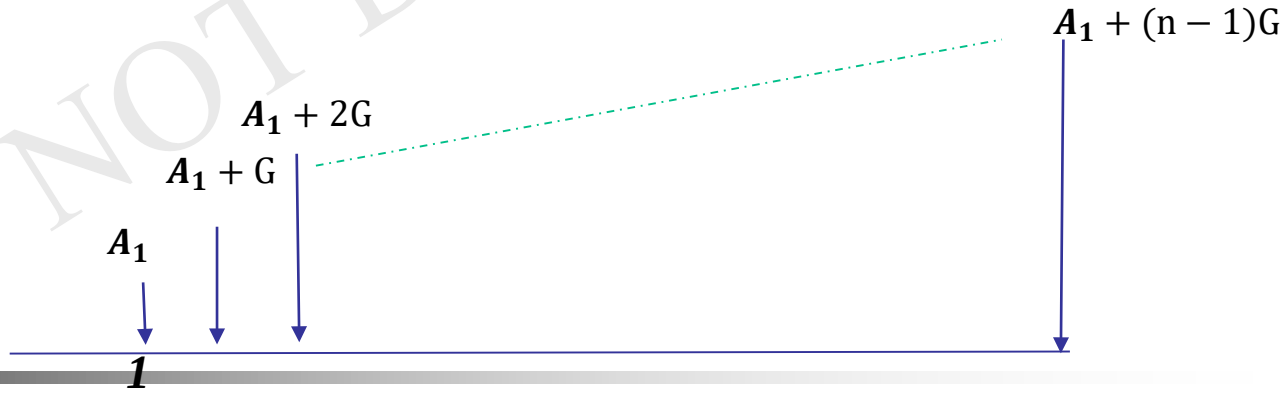
Note: If the slope in the cash flow decreases, the minus should be used in the formulas.

$$\frac{P_G}{G}(Present\ Worth\ factor) \Longrightarrow P = A_1(\frac{P}{A_1}, i, n) \pm G[\frac{(1+i)^n - in - 1}{i^2(1+i)^n}]$$

$$= A_1(\frac{P}{A_1}, i, n) \pm G(\frac{P}{G}, i, n)$$

$$\frac{A_G}{G}(Uniform\ Series) \Longrightarrow A = A_1 \pm G\left(\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right) = A_1 + G\left(\frac{A}{G}, i, n\right)$$

A_1 for the first year

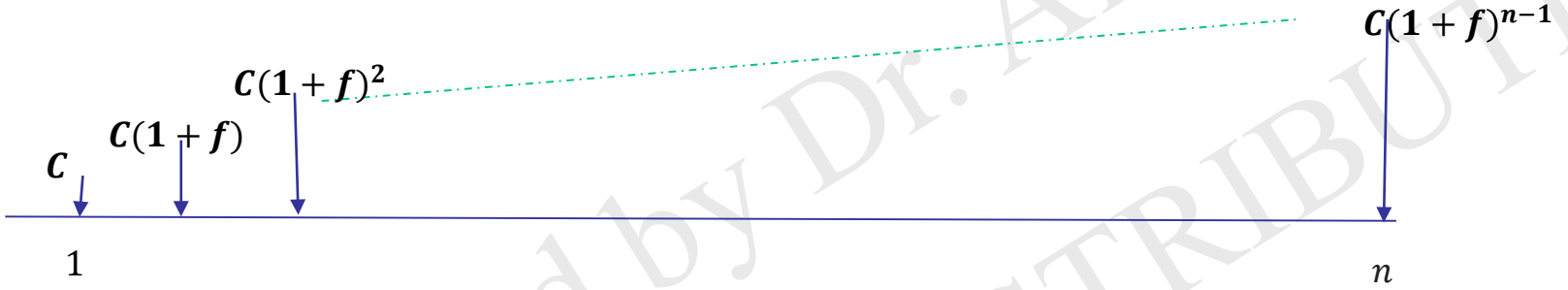


Compound interest – Geometric gradient series factors – Scenario 1

FIND/GIVEN

P/C

(Present Worth factor)



$$\textbf{when } f > i \longrightarrow P = \frac{C}{1+i} * \left(\frac{F}{A}, W, n\right); \textbf{where} \quad W = \frac{1+f}{1+i} - 1$$

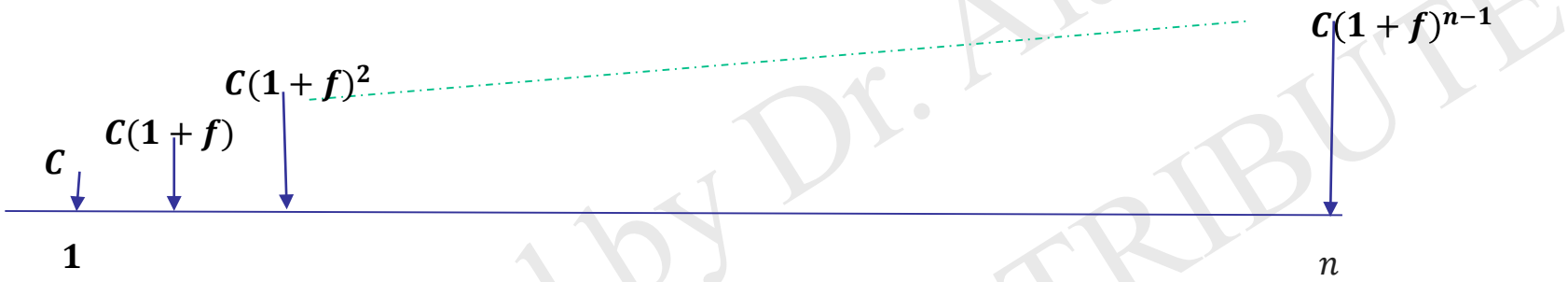
$$A = \frac{C}{1+i} * \left(\frac{F}{A}, W, n\right)\left(\frac{A}{P}, i, n\right); \textbf{where} \quad W = \frac{1+f}{1+i} - 1$$

Compound interest – Geometric gradient series factors – Scenario 1

FIND/GIVEN

P/C

(Present Worth factor)



$$\textbf{when } f < i \longrightarrow P = \frac{C}{1+f} * \left(\frac{P}{A}, W, n\right); \textbf{ where } \quad W = \frac{1+i}{1+f} - 1$$

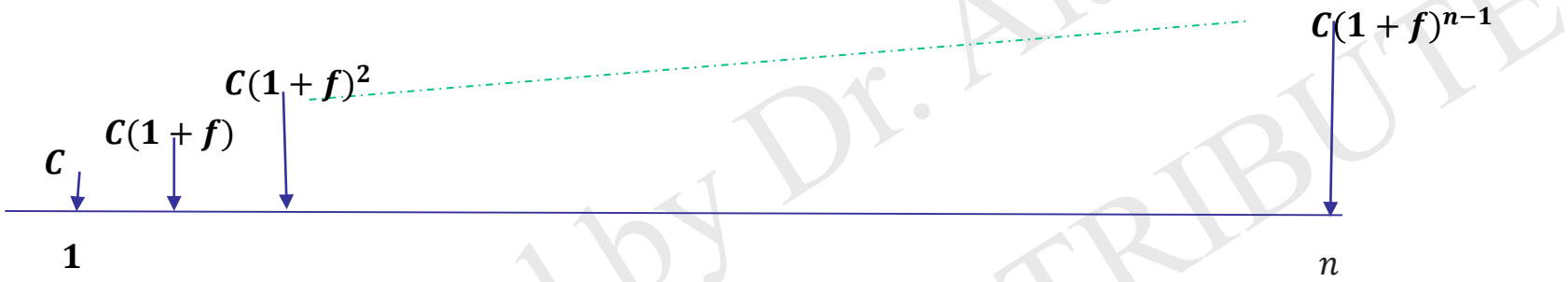
$$A = \frac{C}{1+f} * \left(\frac{P}{A}, W, n\right)\left(\frac{A}{P}, i, n\right); \textbf{ where } \quad W = \frac{1+i}{1+f} - 1$$

Compound interest – Geometric gradient series factors – Scenario 1

FIND/GIVEN

P/C

(Present Worth factor)



$$when\ f = i \longrightarrow P = \frac{nc}{1+f}$$

$$A = \frac{nc}{1+f} * \left(\frac{A}{P}, i, n\right)$$

Compound interest – Geometric gradient series factors – Scenario 1

- When $f > i$;
  - $P = (C / (1 + i))\,(F/A, W, n)$; $W = ((1 + f) / (1 + i)) - 1$
  - $A = (C / (1 + i))\,(F/A, W, n)\,(A/P, i, n)$; $W = ((1 + f) / (1 + i)) - 1$

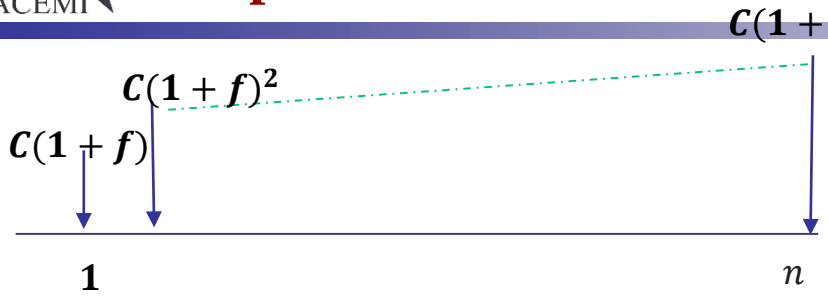
- When $f < i$;
  - $P = (C / (1 + f))\,(P/A, W, n)$; $W = ((1 + i) / (1 + f)) - 1$
  - $A = (C / (1 + f))\,(P/A, W, n)\,(A/P, i, n)$; $W = ((1 + i) / (1 + f)) - 1$

- When $f = i$;
  - $P = n\,(C(1 + f))$;
  - $A = n\,(C / (1 + f))\,(A/P, i, n)$;

- Note: How to interpolate?

$$y = y_1 + \left[\frac{x - x_1}{x_2 - x_1}\right](Y_2 - Y_1)$$

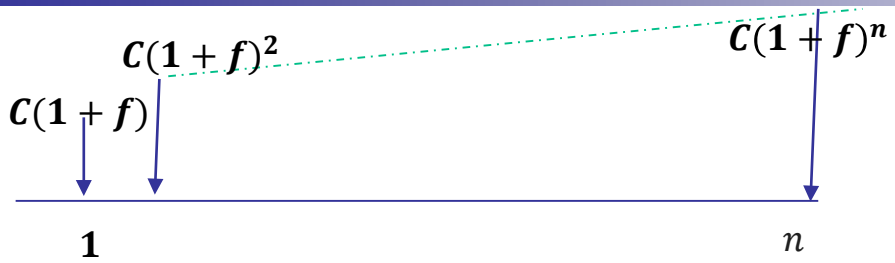
Compound interest – Geometric gradient series factors – Scenario 2



$$\textbf{when } f > i \longrightarrow P = C(1+W)\left(\frac{F}{A}, W, n\right); \textbf{where} \quad W = \frac{1+f}{1+i} - 1$$

$$A = C(1+W)\left(\frac{F}{A}, W, n\right)\left(\frac{A}{P}, i, n\right); \textbf{where} \quad W = \frac{1+f}{1+i} - 1$$

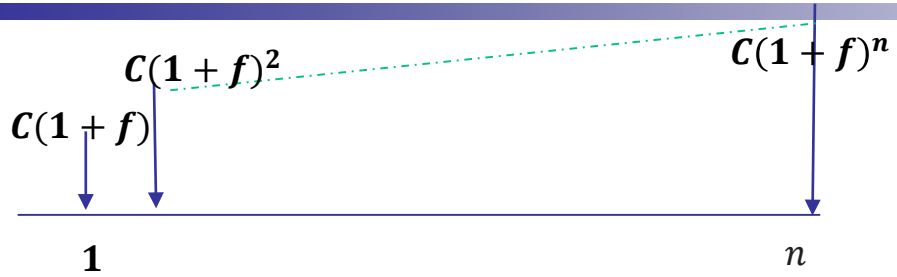
Compound interest – Geometric gradient series factors – Scenario 2



$$\textbf{when } f < i \longrightarrow P = C\left(\frac{P}{A}, W, n\right); \textbf{where} \quad W = \frac{1+i}{1+f} - 1$$

$$A = C\left(\frac{P}{A}, W, n\right)\left(\frac{A}{P}, i, n\right) \longrightarrow w = \frac{1+i}{1+f} - 1$$

Compound interest – Geometric gradient series factors – Scenario 2



$$when\ f = i \longrightarrow P = nc$$

$$A = nc\left(\frac{A}{P}, i, n\right)$$

Compound interest – Geometric gradient series factors – Scenario 2

- When $f > i$;
  - $P = (C\,(1 + W))\,(F/A, W, n)$; $W = ((1 + f) / (1 + i)) - 1$
  - $A = (C\,(1 + W))\,(F/A, W, n)\,(A/P, i, n)$; $W = ((1 + f) / (1 + i)) - 1$

- When $f < i$;
  - $P = C\,(P/A, W, n)$; $W = ((1 + i) / (1 + f)) - 1$
  - $A = C\,(P/A, W, n)\,(A/P, i, n)$; $W = ((1 + i) / (1 + f)) - 1$

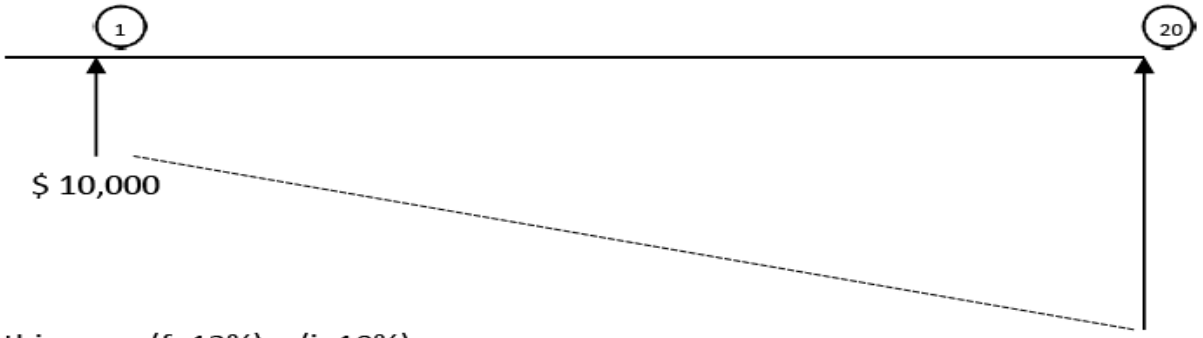
- When $f = i$;
  - $P = n\,C$;
  - $A = n\,C\,(A/P, i, n)$;

Geometric gradient series factors – Example 11

Example 11: The fuel cost for operating a building's heating system for the coming year is expected to be $10,000. The building's estimated life is 20 years and the desired rate of return is 10%. The price of fuel is anticipated to increase at a rate of 12% per year in the remaining 19 years.

a) What is the annual equivalent cost of fuel?

a)



In this case; (f=12%) > (i=10%)

So: $W = \frac{1+f}{1+i} - 1$

$W = \frac{1+0.12}{1+0.10} - 1$

$= 0.018$

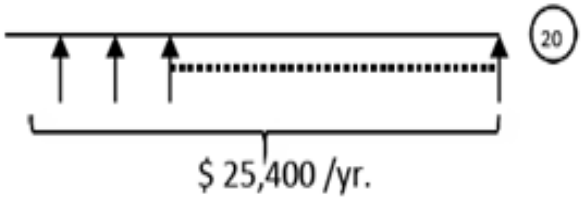
$A = \frac{C}{(1+i)}$ (F/A,W,n) (A/P,i,n)

$A = \frac{10,000}{(1+0.10)}$ (F/A,1.8%,20) (A/P,10,20)

$= \frac{10,000}{1.10}\left[\frac{(1+0.018)^{20}-1}{0.018}\right] \times 0.1175$

= $ 25,400 per year.

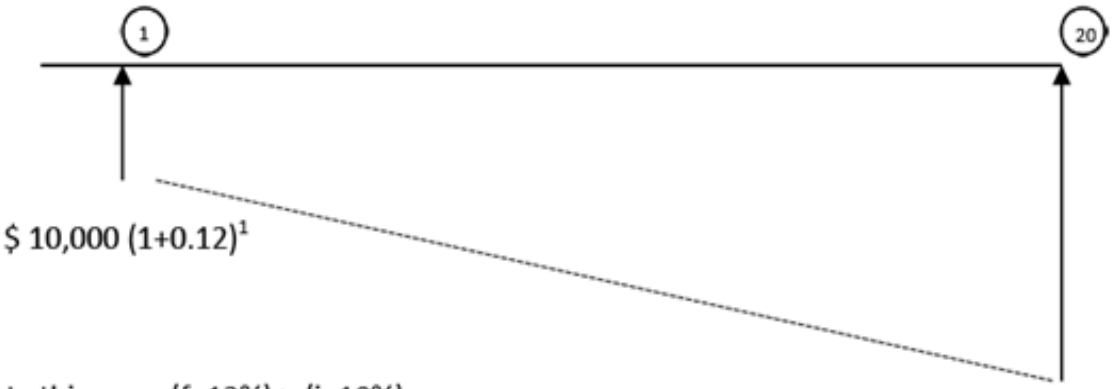
≈



Geometric gradient series factors – Example 11

Example 11: b) What is the annual equivalent cost of fuel if the present day cost is $10,000 and if the price of fuel increases at a rate of 12% per year in the coming 20 years? (Assume again year-end payments).

b)



In this case; (f=12%) > (i=10%)

So: $W = \frac{1+f}{1+i} - 1$

$W = \frac{1+0.12}{1+0.10} - 1$

$= 0.018$

A = C (1+W) (F/A,W,n) (A/P,i,n)

= 10,000 (1+0.018) (F/A,1.8%,20) (A/P,10,20)

$=10{,}000 \times 1.018 \times \left[\frac{(1+0.018)^{20}-1}{0.018}\right] \times 0.1175$

= $ 28,491.48 per year.

Solve problems using EXCEL

In **economic analysis spreadsheets** may be **helpful for a number of purposes**:

1.Constructing tables of cash flows

2.Using annuity **functions** to **calculate *P, F, A, n, or i.***

3.Using a **block function** to find the **present worth** or **internal rate of return** for a table of **cash flows**

4.Making **graphs for analysis** and **presentations**

5.Calculating **"what-if"** for various problem variables.

Nominal and effective interest rate statement

- Here we discuss ***nominal*** *and* ***effective*** ***interest rates***, which have the **same basic relationship**.
  - The **difference** here is that the **concepts** of **nominal** and **effective** must be used when **interest** is **compounded more than once each year**.
    - ✓ For example, if an interest rate is expressed as **1% per month**, the terms ***nominal*** and ***effective*** interest rates **must be considered**.

The interest amounts for loans, mortgages, bonds, and stocks are commonly based upon interest rates compounded more frequently than annually.

Nominal and effective interest rate statement

- An **effective interest rate** ***i*** is a rate wherein the **compounding of interest is taken into account**.

- **Effective rates** are **commonly** expressed on an **annual basis** as an **effective annual rate**; **however**, **any time basis** may be **used**.

Nominal and effective interest rate statement

- r_t = nominal interest rate per year compounded every "t" periods
(نرخ بهره اسمی سالانه)

$$r_t = t \times i_t$$

- t = number of interest periods per year
(تعداد دوره های ترکیب در سال)

- i_t = effective rate of interest per compounding period
(نرخ بهره موثر در هر دوره ترکیب)

- Effective rate of interest of 2% per quarter = nominal rate of interest of 8% per year compounded quarterly

Nominal and effective interest rate statement

- r_t = nominal interest rate per year compounded every "t" periods
- t = number of interest periods per year
- i_t = effective rate of interest per compounding period

$$r_t = t \times i_t$$

Note:

r_t is always expressed **per year** compounded in t and it means r is expressed annually which should be compounded in t times

Nominal and effective interest rate statement

Correct	Incorrect
$r_s = 2i_s$	$r_s = 2i_q$
$r_q = 4i_q$	$r_s = 6i_m$
$r_m = 12i_m$	$r_s = 26i_w$
$r_w = 52i_w$	$r_q = i_s/2$
$r_d = 365i_d$	$r_q = 3i_m$

Nominal and effective interest rate statement

- ***CP*** is compounding period per payment periods
- ***APR*** is annual percentage rate which is equal to r_t $r_t = t \times i_t$
- ***APY*** is annual percentage yield which is equal to ***i***
  - We need to calculate ***i***

Nominal and effective interest rate statement

- r_t = nominal interest rate per year compounded every "t" periods
- t = number of interest periods per year
- i_t = effective rate of interest per compounding period

i = effective equivalent annual interest rate

- Always $r_t < i$

$$r_t = t \times i_t$$

$$i = (1+\frac{r_t}{t})^t - 1 = (1+i_t)^t - 1$$

Nominal and effective interest rate statement

- r = 6% per year compounded annually – i = 6% per year – i = 6.000% per year
- r_s = 6% per year compounded semiannually – i_s = 3% per half year – i = 6.090% per year
- r_q = 6% per year compounded quarterly – i_q = 1.50% per quarter – i = 6.136% per year
- r_m = 6% per year compounded monthly – i_m = 0.50% per month – i = 6.168% per year
- r_w = 6% per year compounded weekly – i_w = 0.0116% per week- i = 6.180% per year
- r_d = 6% per year compounded daily – i = 3% per day – i = 6.183% per year

$$r_t = t \times i_t$$

$$i = (1 + \frac{r_t}{t})^t - 1 = (1 + i_t)^t - 1$$

Method of solving problems

- We have **three types of problem:**

1. Compounding periods and payments coincide
   - (دوره های پرداخت بهره با دوره های پرداخت پول تقارن دارند)

2. Compounding periods more frequent than payments
   - (دوره های پرداخت بهره بیشتر از دوره های پرداخت پول است)

3. Compounding periods less frequent than payments
   - (یعنی دوره های پرداخت بهره کمتر از دوره های پرداخت پول است)

Compounding periods and payments coincide

1. Find **effective interest rate**: $i_t = r_t / t$
2. Find **number of interest periods**: $n_t =$ ***(t)(n*** (number of years))
3. Use **formulas**

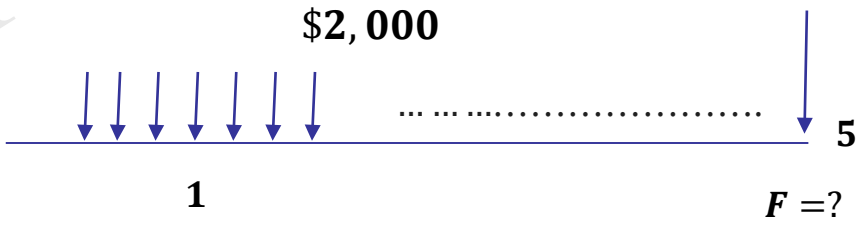
Compounding periods and payments – Example 12

Example 12: Deposit $2,000 quarterly into a fund that pays a nominal interest rate of 12% per year compounded quarterly.

What is the amount accumulated at the end of five years?

$$r_q = 12\%\ per\ year \Rightarrow i_q = \frac{r_q}{4} = 3\% \Rightarrow n_q = 4 * 5 = 20\ quarters$$

$$\Rightarrow n_q\ \&\ r_q\ are\ in\ the\ same\ time \Rightarrow F = \$2000\left(\frac{F}{A}, 3\%, 20\right) = \$53740$$



Compounding periods more frequent than payments

1. Find $i = (1 + i_t)^t - 1$
2. Find $i_p, i = (1 + i_p)^t - 1$
3. Use formulas for $i_p = n_p$

Compounding periods more frequent than payments – Example 13

Example 13: Deposit $2,000 semiannually into a fund that pays a nominal interest of 12% per year compounded quarterly. What is the amount accumulated at the end of five years?

$$r_q = 12\% \Rightarrow i = (1 + \frac{r_q}{4})^4 - 1 = (1 + \frac{0.12}{4})^4 - 1 = 12.55\%$$

$$i_p = i_s \Rightarrow i = (1 + i_p)^2 - 1 = (1 + i_s)^2 - 1 \Rightarrow 0.1255 = (1 + i_s)^2 - 1$$

$$\Rightarrow i_s = 6.10\% \; per\ half\ year \;\; n_s = 2 * 5 = 10 \; half\ years$$

$$or\ (1 + i_s)^2 - 1 = (1 + r_q/4)^4 - 1 \Rightarrow i_s = 6.10\%$$

$$F = A\left(\frac{F}{A}, i_s, n_s\right) = \$2000\left(\frac{F}{A}, 6.1\%, 10\right) = \$26,490$$

Compounding periods less frequent than payments

There are two forms:

- Inter-period interest compounded: yes (if there is interest between the date of payments)

1. Find $i = (1+i_t)^t - 1$
2. Find $i_p, i = (1+i_p)^t - 1$
3. Use formulas for $i_p = n_p$

- Inter-period interest compounded: no (if there is no interest between the date of payments)

1. All receipts at end of compounded period (no interperiod interest is earned)
2. All disbursements at start of compounded period (no interest is paid for interperiod)
3. Use formulas

Compounding periods less frequent than payments – Example 14

Example 14: Deposit $2,000 monthly into a fund that pays a nominal interest rate of 12% per year compounded quarterly.

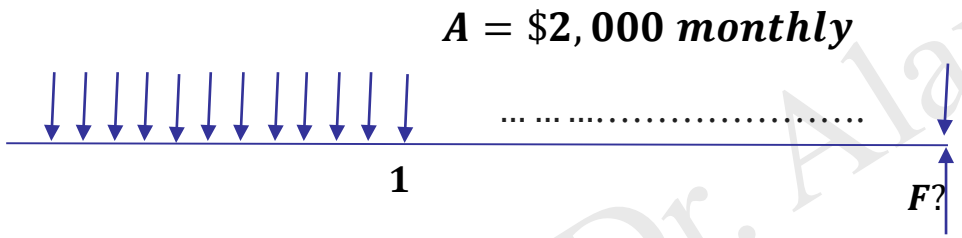
What is the amount accumulated at the end of five years?

a) Assume interperiod interest is compounded

b) Assume interperiod interest is not compounded.

Compounding periods less frequent than payments – Example 14

a) Assume interperiod interest is compounded



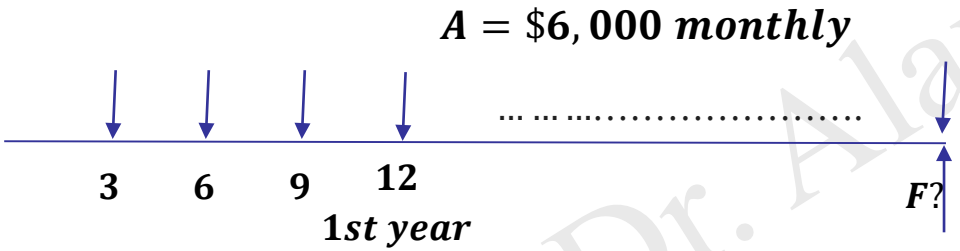
$$r_q = 12\% \Rightarrow i = (1 + \frac{0.12}{4})^4 - 1 = 12.55\%$$

$$i_p = i_m \Rightarrow 12.55 = (1 + i_m)^{12} - 1 \Rightarrow i_m = 1.00\% \Rightarrow n_m = 12 * 5 = 60\ months$$

$$F = \$2000\left(\frac{F}{A}, 1\%, 60\right) = \$163340$$

Compounding periods less frequent than payments – Example 14

b) Assume interperiod interest is not compounded.



$$r_q = 12\% \Rightarrow i_q = 3\% \Rightarrow n_q = 20\ quarters \Rightarrow F = A\left(\frac{F}{A}, 3\%, 20\right) = \$161,220$$

رویکردی جامع برای ارزیابی اقتصادی در پروژه
همراه با آموزش مدلسازی در اکسل (Excel)

بخش چهارم:

ارزیابی اقتصادی: تصمیم گیری بین گزینه های مختلف سرمایه گذاری

ارائه دهندگان:

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What we will learn?

- Types of alternatives
- How to use quantitative methods
- Methods of comparison between alternatives
- Sunk costs
- Minimum attractive rate of return (MARR)
- Present worth (PW) or net present value (NPV)
- Perpetual life
- Annual equivalent
- Rate of return (ROR)
- Internal rate of return (IRR)
- Incremental rate of return
- External rate of return (ERR)
- Payback period without interest
- Payback period with interest
- Project balance
- Study period
- Unequal lives

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Types of Alternatives

- **Financial decisions** can be **viewed** as the **selection of one or more alternatives** from a **pool of alternatives**.

1. **Independent alternatives** (مستقل)
2. **Dependent alternatives** (وابسته)
   - **Mutually exclusive alternatives** (ناسازگار)
   - **Contingent alternatives** (مشروط)
3. **“Do nothing”**

Types of Alternatives – Independent alternatives

- **Acceptance** of one alternative does **not preclude** the **selection** of the other alternatives.
  - For example, **HVAC** and **marketing** projects.
  - We can choose alternative A or Alternative B or alternative A+B
    - ✓ Due to **sufficient capital**, you can invest in **both of these alternatives**.

Few companies encounter independent alternatives

because

most of alternatives affect others and are not inherently independent

Types of Alternatives – Dependent alternatives

1. **Mutually exclusive alternatives** (Our examples in this course) (پیشنهادهای ناسازگار)

- **Acceptance** of **one** of the alternatives **precludes** investing in the **other alternatives**.

- For example: we need to purchase a **single loader** and has narrowed the selection down to **three loaders**:
  - ✓ When one of the **loaders** purchased, the **need** for a **loader** has been **fulfilled**.

Types of Alternatives – Dependent alternatives

2. **Contingent alternatives** (پیشنهادهای مشروط)

- Very **complex** alternatives
- **Must** be selected with **another alternative**
  - For example, if **your company** is looking at purchasing a **dump truck** and as a **second alternative** purchasing a **dumping trailer** to be pulled **behind** the dump truck:
    - ✓ **Dumping trailer** is a **contingent alternative** because you **must first** purchase the **dump truck**.

Types of Alternatives – Do nothing

- Decisions are **limited** by
  1. **Types of alternatives** in the pool
  2. **External constraints**
     - A **common external constraints are**:
       1. *Limited supply of capital*
       2. *Contractual obligations*
          - ✓ If a company has a contract to perform **excavation work** that requires the use of a **special loader attachment** which the company **does not own**—management must **make a decision** that makes the attachment available for use **on the contract**.

When making decisions, both the external constraints and available alternatives must be taken into account

How to use quantitative methods?

- **To use** the **quantitative methods**:
  1. Deciding in a **single alternative**
  2. Deciding between **mutually exclusive alternatives**
  3. Deciding between **contingent** or **independent alternatives**

How to use quantitative methods? – Single alternative

1. Deciding in a single alternative:

- ✓ Investment in the **alternative** or **"do nothing"**?
  - o Must decide if the alternative meets the **minimum requirement** for **profitability**.

How to use quantitative methods? – Mutually exclusive alternatives

2. Deciding between mutually exclusive alternatives:

- Which of the alternatives is **most financially advantageous** while meeting the **external constraints**.
  - ✓ If investment in **none of the alternatives** is an option, the **"do nothing"** alternative should be included in the **pool of alternatives**.

How to use quantitative methods? – Contingent or independent alternatives

3. Deciding between contingent or independent alternatives:

- Decision must be **reduced** to a set of **mutually exclusive alternatives** **before** the alternative may be **analyzed quantitatively**.
  - ✓ This is done by **listing all** of the **possible combinations** of alternatives and **eliminating** **infeasible alternatives**.
    - *A combination of alternatives* is ***infeasible*** *when:*
    1. It contains **two or more mutually exclusive alternatives**
    2. It Contains a **contingent alternative** **without** **containing** the **alternative** that the **contingent alternative** is **contingent on**
    3. It does **not meet** the **requirements** of the **external factors**

How to convert alternatives to mutually exclusive alternatives? – Example 1

Example 1: A manager has up to $200,000 available to invest in new construction equipment for a construction company.

The potential new construction equipment has been reduced to the alternatives shown in Table.

The manager must purchase a backhoe to complete an existing contract.

A second backhoe is not needed.

Prepare a list of the mutually exclusive alternatives and identify which alternatives are not feasible.

Alternatives

No.	Description	Cost ($)
1	Backhoe 1	100,000
2	Backhoe 2	120,000
3	Dump Truck	65,000
4	Dumping Trailer for the Dump Truck	25,000

How to convert alternatives to mutually exclusive alternatives? – Example 1

Example 1: A manager has up to $200,000 available to invest in new construction equipment for a construction company. The potential new construction equipment has been reduced to the alternatives shown in Table. The manager must purchase a backhoe to complete an existing contract. A second backhoe is not needed. Prepare a list of the mutually exclusive alternatives and identify which alternatives are not feasible.

Alternatives

No.	Description	Cost ($)
1	Backhoe 1	100,000
2	Backhoe 2	120,000
3	Dump Truck	65,000
4	Dumping Trailer for the Dump Truck	25,000

Solution: Two external constraints:

1. $200,000 for investment
2. Only one backhoe is needed.

- 1 and 2 = Mutually exclusive
- 1 and 3 = Independent
- 2 and 3 = Independent
- 4 = Contingent on 3
- Combinations of alternatives that do not include 1 or 2 are unacceptable.
- Combinations of alternatives that include both 1 and 2 are unacceptable.
- Combinations of alternatives that include 4 but do not include 3 are unacceptable.
- Combinations of alternatives that cost more than $200,000 are unacceptable.

How to convert alternatives to mutually exclusive alternatives? – Example 1

Solution: Two external constraints:

1. $200,000 for investment
2. Only one backhoe is needed.

- 1 and 2 = Mutually exclusive
- 1 and 3 = Independent
- 2 and 3 = Independent
- 4 = Contingent on 3
- Combinations of alternatives that do not include 1 or 2 are unacceptable.
- Combinations of alternatives that include both 1 and 2 are unacceptable.
- Combinations of alternatives that include 4 but do not include 3 are unacceptable.
- Combinations of alternatives that cost more than $200,000 are unacceptable.

These five combinations now may be quantitatively analyzed.

Combination of Alternatives

Alternatives	Acceptable	Reasoning
Do nothing	No	Does not contain 1 or 2
1	Yes	
2	Yes	
3	No	Does not contain 1 or 2
4	No	Does not contain 1 or 2
1 and 2	No	Selects both 1 and 2
1 and 3	Yes	
1 and 4	No	Selects 4 without 3
2 and 3	Yes	
2 and 4	No	Selects 4 without 3
3 and 4	No	Does not contain 1 or 2
1, 2 and 3	No	Selects both 1 and 2
1, 2 and 4	No	Selects both 1 and 2
1, 3 and 4	Yes	
2, 3 and 4	No	Exceeds budget
1, 2, 3 and 4	No	Selects both 1 and 2, exceeds budget

Methods of comparison between alternatives

Methods:

1. Present worth (PW) (ارزش خالص فعلی) or net present value (NPV) == Basis of comparison == **Size of cash flow**
2. Future worth (FW) (ارزش خالص آتی) == Basis of comparison == **Size of cash flow**
3. Annual equivalent (AE) (ارزش معادل سالانه) == Basis of comparison == **Size of cash flow**
4. Rate of return (R of R) (ROR) (نرخ بازده سرمایه) == Basis of comparison == **Interest rate**
5. Internal rate of return (IRR) (نرخ بازده داخلی) == Basis of comparison == **Interest rate**
6. Incremental rate of return (نرخ بازده افزایشی) == Basis of comparison == **Interest rate**
7. External rate of return (ERR) (نرخ بازده خارجی) == Basis of comparison == **Interest rate**
8. Payback period (دوره بازپرداخت) == Basis of comparison == **Time**
9. Project balance (تراز پروژه) == Basis of comparison == **Size of cash flow**

Methods of comparison between alternatives

Assumptions:

- Cash flows reasonably accurate
- No inflation
- Interest rate reasonably well-known
- Before tax
- No intangible factors
  - Only quantitative factors

Sunk costs (هزینه بدون بازگشت)

Sunk costs are:

- Costs already **incurred**
- Costs that have been **committed** to be paid that **cannot be canceled**
- Costs that **should not** be used in **decision making** because the money **has already been spent**

Minimum attractive rate of return (حداقل نرخ بازگشت جذاب)

- **Interest rate** at which alternatives will **be evaluated**.
- The **MARR** is the **lowest rate of return** or **interest paid** by an **investment** that is **acceptable** to the **investor**.
- The **MARR** is **different** for **each company**.
- **How to calculate MARR?**

Cost of capital + minimum acceptable profit + Risk

Minimum attractive rate of return – Cost of capital

Cost of capital + minimum acceptable profit + Risk

❑ Cost of capital:

- ✓ **Cost of capital** is **difficult** to calculate if **interest rates are variable**, which should include **any anticipated changes** in the **interest rate.**

1. **For a company that has to borrow money to make investments:**
   - ✓ **Cost of capital** = **effective annual interest rate** at which the money will have to be **borrowed**.
2. **For a company that will use existing funds to make investments:**
   - ✓ **Cost of capital** = **effective annual interest rate paid on the deposit**
     - o Because of the **opportunity cost**.
   - ✓ If **effective annual interest rate** is used,
     1. **Inflation** should be **subtracted** or
     2. **All costs** and **incomes** should **include inflation**.

Minimum attractive rate of return (حداقل نرخ بازگشت جذاب)

Cost of capital + minimum acceptable profit + Risk

❑ Minimum acceptable profit:

- ❖ **Profit level** is quite **subjective** and is often **set** by **management**.
- ❖ **High MARR** will result in the company **passing up many good investments**.
- ❖ **Low MARR** will result in the company making **too many bad or marginal investments**.

Study period

- The **period of time** over **which the alternatives** will be **studied**
- All alternatives **must** have the **same life span** as the **study period**.
- There can be **two conditions:**
  1. Equal life
  2. Unequal life
     - Should be adjusted.

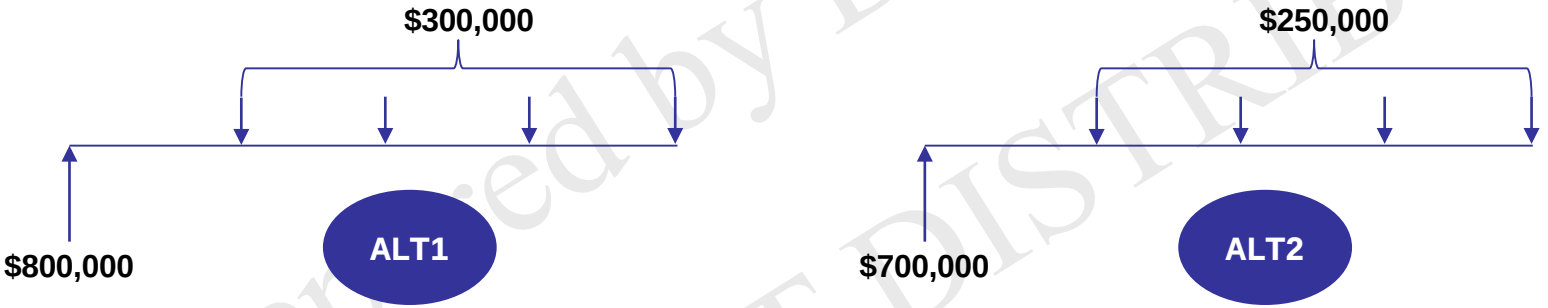
Equal life - PW or NPV

1. **Determine present value** (at beginning of the study period) for **each alternative's cash flows**
2. **Select alternative with largest NPV**
   - If:
   1. $PW_{income}(MARR) - PW_{expenditure}(MARR) > 0$ or $NPV > 0$:
      - Alternative has return > MARR
      - Good
   2. $PW_{income}(MARR) - PW_{expenditure}(MARR) = 0$ or $NPV = 0$:
      - Alternative has return = MARR
      - Do nothing
   3. $PW_{income}(MARR) - PW_{expenditure}(MARR) < 0$ or $NPV < 0$:
      - Alternative has return < MARR
      - Not desirable

Equal life - PW or NPV – Example 1

Example 1:

Alternative (1) has a first cost of $800,000 and an annual income of $300,000.
Alternative (2) has a first cost of $700,000 and an annual income of $250,000.
Take a study period of 4 years and a MARR of 10%.
Which alternative is economically preferable? Use the Present worth Method of comparison.



ALT3: Do Nothing

Equal life - PW or NPV – Example 1

$$MARR = 10\%$$

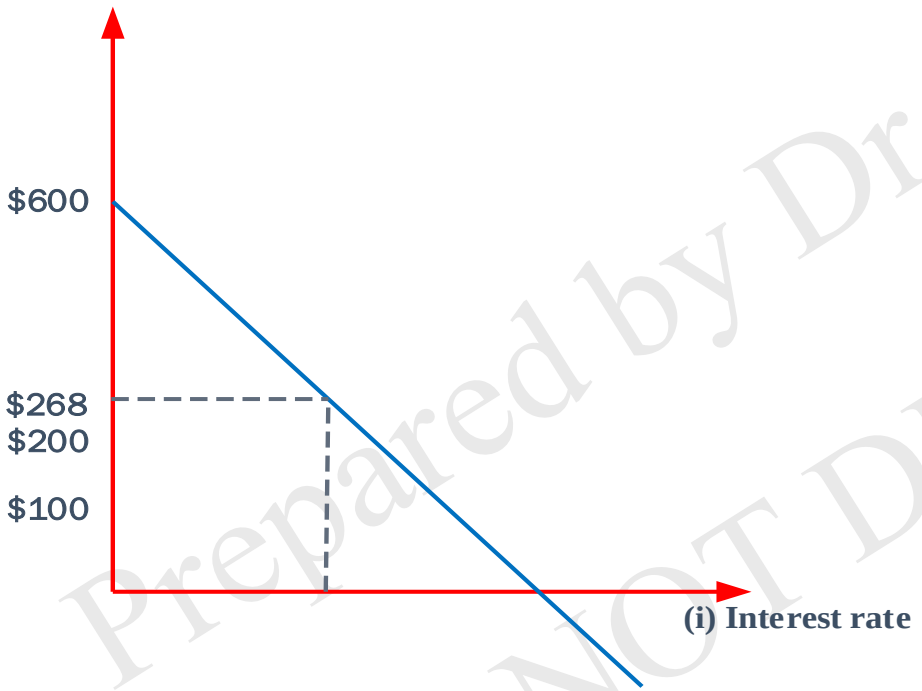
$$PW_1(10\%) = \$300,000\left(\frac{P}{A}, 10\%, 4\right) - \$800,000 = \$151,000$$

$$PW_2(10\%) = \$250,000\left(\frac{P}{A}, 10\%, 4\right) - \$700,000 = \$92,500$$

$$PW_1 > PW_2$$

Equal life - PW or NPV – Example

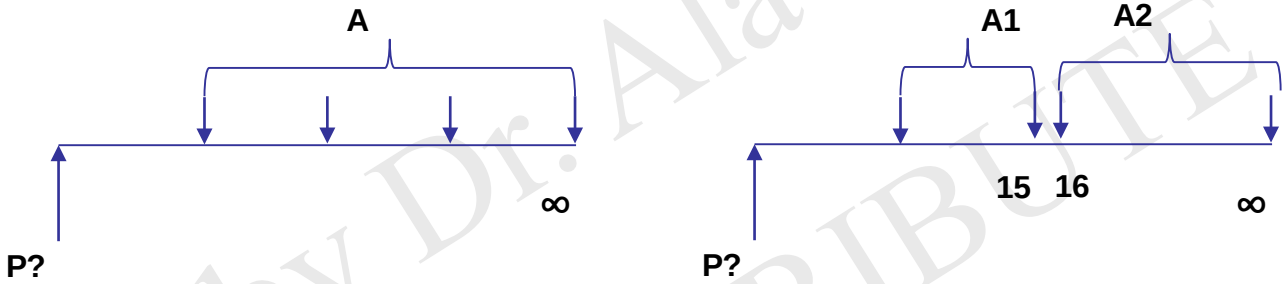
- علاوه بر برگشت $268 در زمان حال به برگشت 10% روی تراز سرمایه گذاری شده می انجامد.



Perpetual life (مادام العمر)

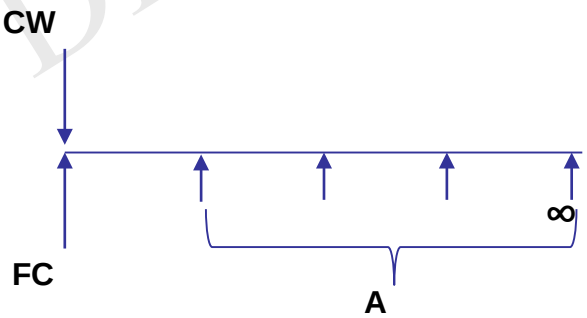
1. **How much** should we **invest now (P)** to achieve **annual income (A) perpetually**:

   - ***P = A (1/i)***



2. **How much** should we **invest now (CW)** to pay the **first cost (FC)**

   - ***CW = FC + A (1/i)***



Perpetual life (مادام العمر) – Example 2

Example 2: A $500,000 gift was given to a city for the construction and continued upkeep of a music hall.

Annual maintenance for the hall is estimated at $15,000.

In addition, $25,000 will be needed every 10 years for painting and major repairs.

Deposited funds can earn 6% annual interest and the returns are not subject to taxes.

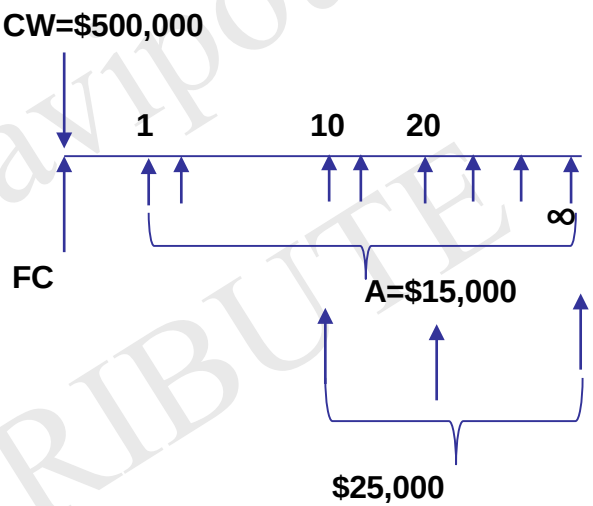
How much will be left for the initial construction costs after funds are allocated for perpetual upkeep?

Perpetual life (مادام العمر) – Example 2

Note: we have to convert $25,000/year to annual amount

$$FC = CW - A(\frac{1}{i})$$

$$FC = \$500,000 - \frac{\$15,000}{0.06} - 25,000\left(\frac{A}{F}, 6\%, 10\right) * \frac{1}{0.06} = \cdots$$



Equal life – AE

1. Determine **annual equivalent** for **each alternative's cash flows**
2. Select alternative with **largest AE**
   - If:
   1. $AE_{income}(MARR) - AE_{expenditure}(MARR) > 0$:
      - Alternative has return > MARR
      - Good
   2. $AE_{income}(MARR) - AE_{expenditure}(MARR) = 0$:
      - Alternative has return = MARR
      - Do nothing
   3. $AE_{income}(MARR) - AE_{expenditure}(MARR) < 0$:
      - Alternative has return < MARR
      - Not desirable

Equal life – AE – Example 3

Example 3:

Alternative (1) has a first cost of $800,000 and an annual income of $300,000.

Alternative (2) has a first cost of $700,000 and an annual income of $250,000.

Take a study period of 4 years and a MARR of 10%. Which alternative is economically preferable? Use the Present worth Method of comparison.

$$AE_1(10\%) = \$300,000 - \$800,000\left(\frac{A}{P},10,4\right) = \$47,600\ per\ year$$

$$AE_2(10\%) = \$250,000 - \$700,000\left(\frac{A}{P},10,4\right) = \$29,150\ per\ year$$

$\Rightarrow select\ 1$

Equal life - Comparison between PW, AE, and FW

AE selects the same alternative as the PW (NPV) and FW

$$\frac{PW(MARR)_A}{PW(MARR)_B} = \frac{AE(MARR)_A}{AE(MARR)_B} = \frac{FW(MARR)_A}{FW(MARR)_B}$$

Equal life - Rate of return (ROR)

- **Rate of return** is also known as the **return on investment**.

1. **From the perspective of lenders:**
   - It is the **interest rate paid** on the **unrecovered balance** at **each time period**.
2. **From the perspective of borrowers:**
   - It is the **interest rate paid** on the **unpaid balance** of **borrowed money**.
3. **From the perspective of investors:**
   - It is the **interest rate paid** on the **unrecovered balance** of an **investment** over the **remaining life** of the **investment**.

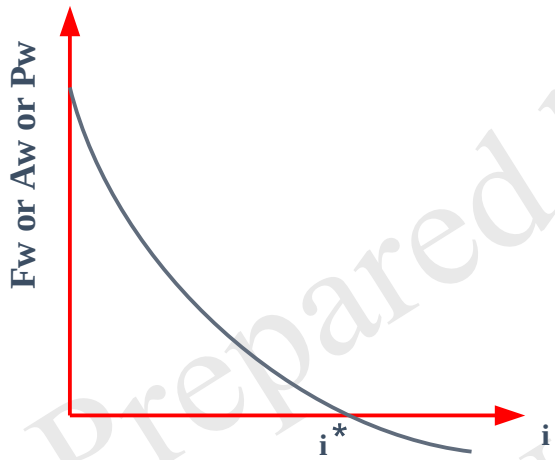
Equal life - Rate of return (ROR)

1. **One rates of return (ROR)**
   - ❖ It means, the **cumulative cash flow** does **not switch** between **– (negative) and + (positive) more than once**.
     - ✓ ***Internal rate of interest (IRR)***

2. **Multiple rates of return:**
   - ❖ It means, the **cumulative cash flow switches** between **– (negative) and + (positive) more than once**.
     - ✓ ***External rate of return (ERR)***

Equal life - Internal rate of return (IRR)

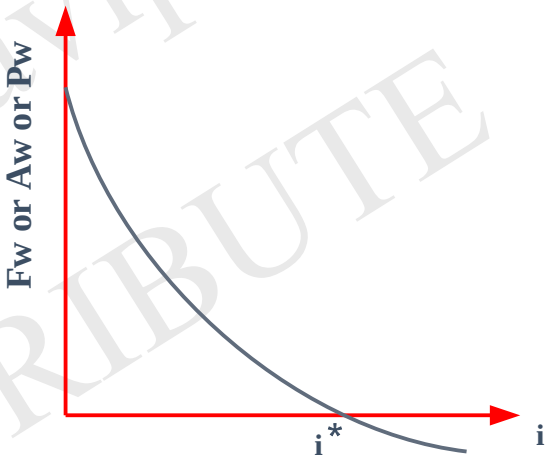
- **How to calculate ROR?**
  - **ROR produces PW (NPV) or AE or FW of zero.**
    - ✓ PW (NPV) = 0 , or AE = 0, or FW = 0 ====== ROR?



Equal life - Internal rate of return (IRR)

- **How to use ROR for making decision between alternatives?**

1. ***ROR > MARR:***
   - PW (NPV), FW, and AE > 0
   - Good
2. ***ROR = MARR:***
   - PW (NPV), FW, and AE = 0
   - Do nothing
3. ***ROR < MARR:***
   - PW (NPV), FW, and AE < 0
   - Not desirable



Select alternative** with **largest rate of return

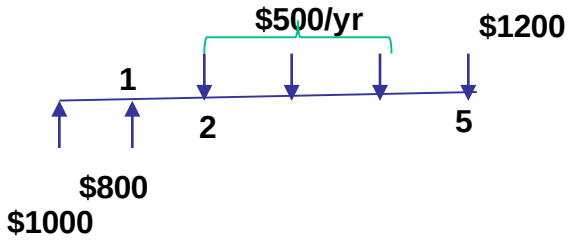
If best rate of return is less than the MARR consider rejecting all alternative

Equal life - Internal rate of return (IRR) – Example 4

Example 4: Find the rate of return of a cash flow where there is a disbursement of $1,000 now, $800 at the end of the first year, followed by $500 annual income at the end of the second, third, and fourth years.

The last income payment of $1,200 takes place at the end of the fifth year.

Equal life - Internal rate of return (IRR) – Example 4



$$PW(i) = 0 \Rightarrow -\$1000 - \$800\left(\frac{P}{F}, i*, 1\right) + \$500\left(\frac{P}{A}, i*, 3\right)\left(\frac{P}{F}, i*, 1\right) + \$1,200\left(\frac{P}{F}, i*, 5\right) = 0 \Rightarrow$$

$$i* = 12.77\%$$

Equal life - Internal rate of return (IRR) – Checking a single alternative

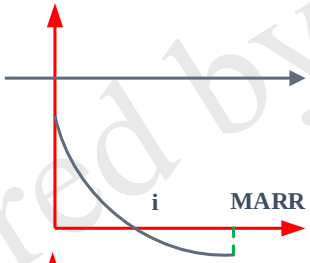
We do not need to calculate IRR to see whether the alternative is good or not.

1. PW (MARR) < 0 ==== MARR > i*
2. PW (MARR) > 0 ==== MARR < i*
3. PW (MARR) = 0 ==== MARR = i*

This can be used to check only one alternative

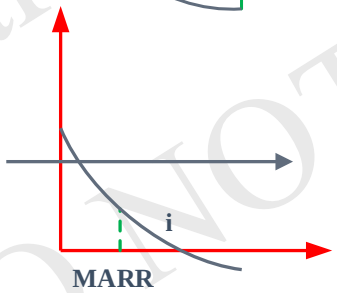
برای محاسبه و مقایسه MARR بدون محاسبه i^*
⇐ اگر از Cash Flow ⇐(CH-CB) ⇐MARR را بگذاریم.

PW (MARR) < 0



MARR > i

PW (MARR) > 0



MARR < i

PW (MARR) = 0 ⟶ $MARR = i$

Equal life - Internal rate of return (IRR)

Note:

We cannot rely only on calculating simply the IRR to make decision between alternatives. Why?

See the example…

Equal life - Internal rate of return (IRR) – Example 5

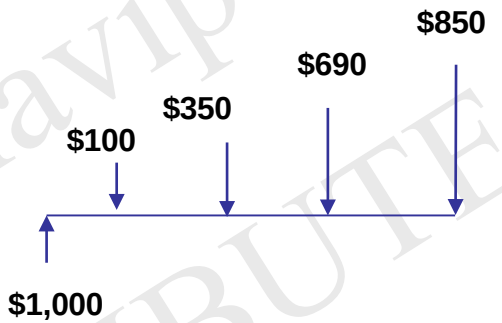
- ***Example 5:*** Two projects have the following cash flows. Which one is economically more attractive? Use the rate of return method of comparison. Take MARR = 10%.

	PROJECT X	PROJECT Y
Now	-$1,000	-$1000
End of 1st year	$100	$1,000
End of 2nd year	$350	$200
End of 3rd year	$600	$200
End of 4th year	$850	$200

Equal life - Internal rate of return (IRR) – Example 5

	PROJECT X	PROJECT Y
Now	-$1,000	-$1000
End of 1st year	$100	$1,000
End of 2nd year	$350	$200
End of 3rd year	$600	$200
End of 4th year	$850	$200

Project X



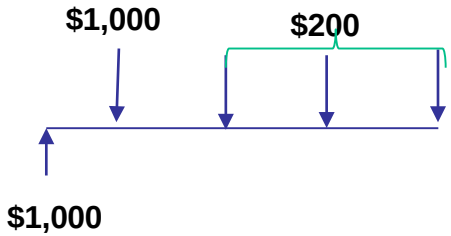
$$PW_x(i*) = -1,000 + \left[100 + 250\left(\frac{A}{G}, i*, 4\right)\right] \times \left(\frac{P}{A}, i*, 4\right)$$

$$\Rightarrow \quad i*_x = 23.4\%$$

$$PW_y(i*) = -1,000 + 1000\left(\frac{P}{F}, i*, 1\right) + 200\left(\frac{F}{A}, i*, 3\right)\left(\frac{P}{F}, i*, 4\right)$$

$$\Rightarrow \quad i*_y = 34.5\%$$

Project Y



X or Y?

Equal life - Internal rate of return (IRR) – Example 6

Example 6: Two projects have the following cash flows. Which one is economically more attractive? Use the rate of return method of comparison. Take MARR = 10%.

	PROJECT X	PROJECT Y
Now	-$1,000	-$1000
End of 1st year	$100	$1,000
End of 2nd year	$350	$200
End of 3rd year	$600	$200
End of 4th year	$850	$200

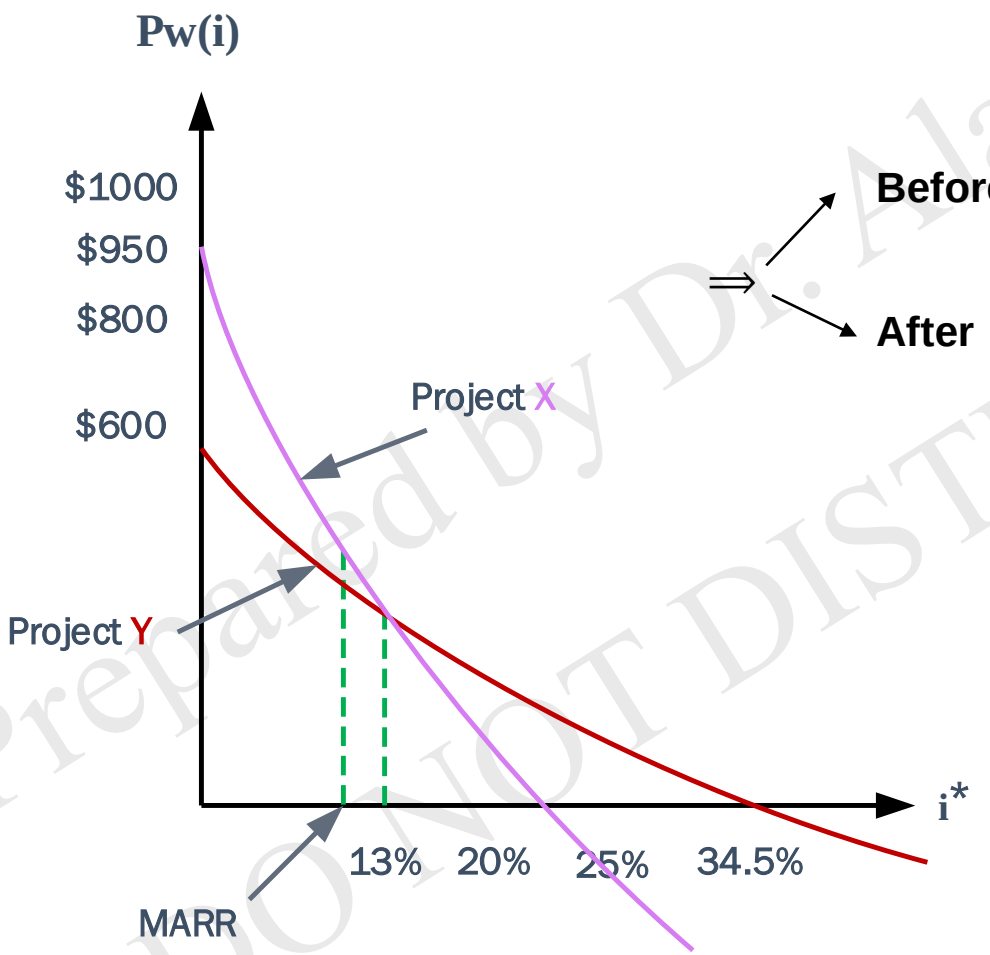
Check the result by MARR(10%):

$$PW_x(10\%) = -\$1,000 + \left[\$100 + \$250\left(\frac{A}{G}, 10, 4\right)\right]\left(\frac{P}{A}, 10, 4\right) = \$412$$

$$PW_y(10\%) = -\$1,000 + \$1,000\left(\frac{P}{F}, 10, 1\right) + \$200\left(\frac{F}{A}, 10, 3\right)\left(\frac{P}{F}, 10, 4\right) = \$361$$

X or Y?

Equal life - Internal rate of return (IRR) – Example 6



Before 13% ⟶ Project X>Project Y

After 13% ⟶ Project Y>Project X

What is the solution?

Equal life - Incremental rate of return – Checking multiple alternatives

The solution is: Incremental rate of return analysis that is used for multiple alternatives:

Rank all of the alternatives in order of **initial cost** from **lowest to highest**.

- If **first costs** are **equal**, it should be based on **ascending** order of (*PW(i = 0)income – PW(i = 0) expenditure*))

1. Select first on the list as "BEST CURRENT"
2. Select next on the list as "CHALLENGER"
3. Find incremental cash flows (CH – BC)
4. Calculate i* for (CH – BC)
5. If i* ≤ MARR then BC remains BC, CH is eliminated
6. If i* > MARR then BC is eliminated, CH remains BC
7. Continue to the end and to the last alternative

Equal life - Incremental rate of return – Checking multiple alternatives

You don't need to calculate i*. Put MARR instead:

1. **If PW(MARR) < 0 ===== MARR > i***
2. **If PW(MARR) > 0 ===== MARR < i***

نیازی به تشخیص i^* به صورت دقیق بلکه با قرار دادن MARR به جای i می توانیم بنویسیم

i^* از $MARR$ بزرگتراست،کوچکتر

PW(MARR) < 0 ⇐ 1- باگذاشتن MARR

MARR > i

PW(MARR) > 0 ⇐ 2- باگذاشتن MARR

MARR < i

Equal life - Incremental rate of return – Example 7

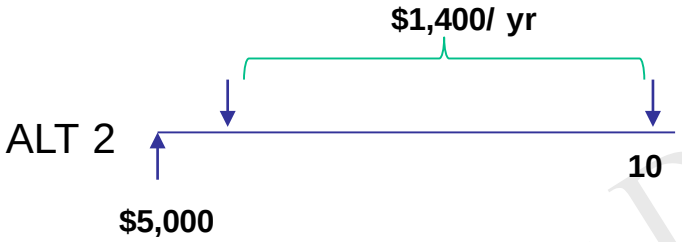
Example 7: There are four mutually exclusive alternatives whose cash flows are given below. Calculate by the rate of return method, which one we should select. Take MARR = 15%.

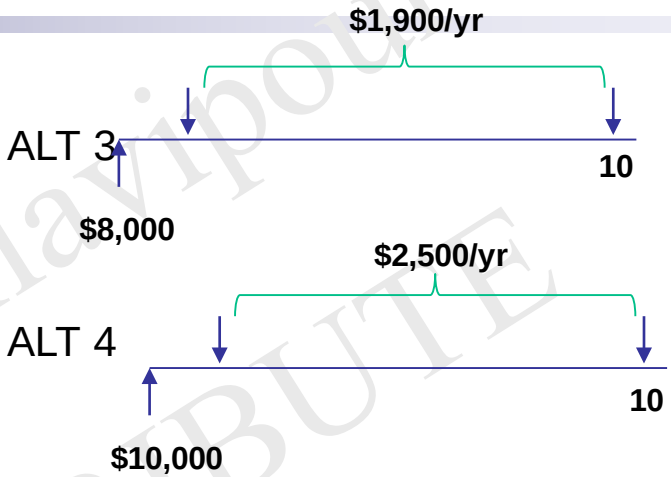
	FIRST COST	ANNUAL INCOME	LIFE
ALTERNATIVE 1	$0	$0	10 YRS
ALTERNATIVE 2	$5,000	$1,400	10 YRS
ALTERNATIVE 3	$8,000	$1,900	10 YRS
ALTERNATIVE 4	$10,000	$2,500	10 YRS

Equal life - Incremental rate of return – Example 7

Solution Example 5

ALT 1 ⟶ Do nothing

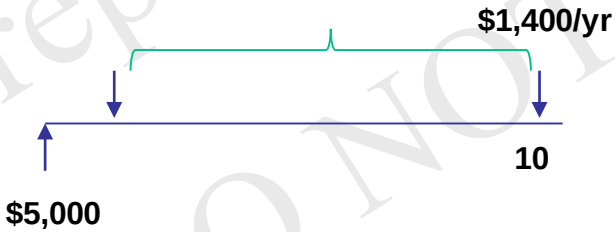




⟶ **MARR =15%**

Ascending order: ALT 1, ALT 2, ALT 3, ALT 4

1. *BC* ⟶ *ALT 1* , *CH* ⟶ *ALT 2*



$$PW_{(2-1)}(i^*) = -\$5,000 + \$1,400\left(\frac{P}{A}, i^*, 10\right) = 0$$

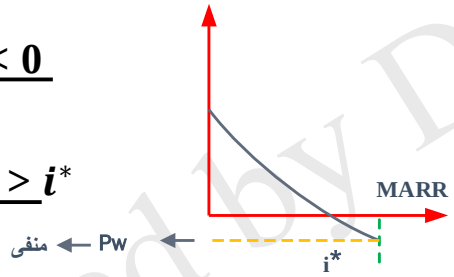
$$\Rightarrow i^* = 25\% > MARR$$

Equal life - Incremental rate of return – Example 7

نیازی به تشخیص i^* به صورت دقیق بلکه با قرار دادن MARR به جای i می توانیم بنویسیم

i^* از $MARR$ بزرگتراست،کوچکتر

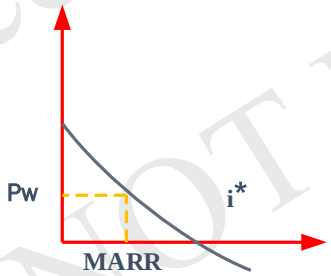
PW(MARR) < 0



⇐ 1- باگذاشتن MARR

MARR > i^*

PW(MARR) > 0



⇐ 2- باگذاشتن MARR

MARR < i^*

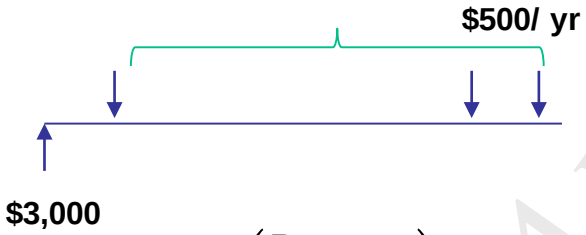
$BC \longrightarrow ALT\,1 \longrightarrow Eliminated$ $\qquad$ $CH \longrightarrow ALT\,2 \longrightarrow Becomes\,BC$

$$PW_{2-1}(15\%) = \$2,027 > 0 \implies i^* > MARR$$

Equal life - Incremental rate of return – Example 7

2. *BC: ALT 2*

CH: ALT 3



$$PW_{(3-2)}(i^*) = -\$3,000 + \$500\left(\frac{P}{A}, i^*, 10\right) = 0 \Rightarrow i = 10.50\% < MARR$$

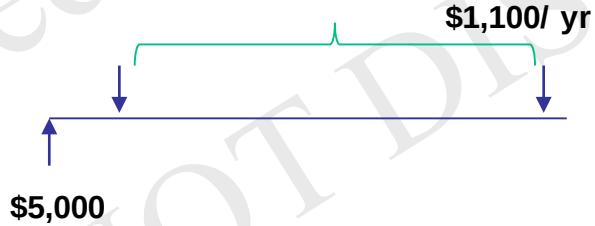
$$OR \;\; PW_{(3-2)}(15\%) = -\$491 > 0 \Rightarrow MARR > i^*$$

BC remains BC ==ALT 2

CH is Eliminated==ALT 3

3. *BC: ALT 2*

CH: ALT 4



$$PW_{(4-2)}(i^*) = -\$5,000 + \$1,100\left(\frac{P}{A}, i^*, 10\right) = 0 \Rightarrow i^* = 17.70\% > MARR$$

BC ⟶ *ALT 2* *is Eliminated*

CH ⟶ *ALT 4* *Becomes* *BC*

$\Rightarrow$ *SELECT 4*

Equal life - External rate of return (ERR)

- **Use when there are multiple rates of return**
  - It means, the cumulative cash flow **switches between – (negative) and + (positive) more than once**.

Equal life - External rate of return (ERR)

- **How to calculate?**
  1. **Find** ***FW*** **receipts** at ***MARR***
  2. **Find** ***FW*** **disbursements** at ***ERR*** **(put passive amount for** ***ERR*****)**
  3. **Make equal** both **receipts and disbursements equations**
  4. **Find** ***ERR*** and if
     - ✓ ***ERR*** **>** ***MARR*** , this is **desirable** investment
     - ✓ ***ERR*** **<** ***MARR***, this is **not desirable** investment

Equal life - External rate of return (ERR) – Example 8

Example 8: One of the alternatives for improving an operation is to do nothing to it for 2 years and then spend $10,000 on improvements.

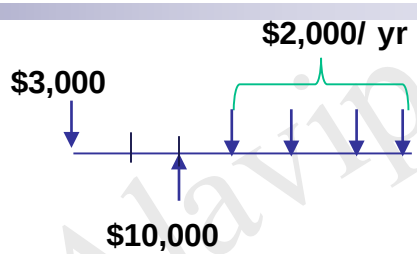
If this course of action is followed, the immediate gain is $3,000 followed by 2 years of breakeven operations.

Thereafter annual income should be $2,000 per year for 4 years.

What rate of return can be expected from following this course of delayed action? MARR = 15%.

Equal life - External rate of return (ERR) – Example 8

End Of Year	Cumulitive Cash flow
0	+3000
1	+3000
2	-7000
3	-5000
4	-3000
5	-1000
6	+1000



$MARR$=15%

$$FW(MARR) = 3{,}000\left(\frac{F}{P}, 15\%, 6\right) + 2{,}000\left(\frac{F}{A}, 15\%, 4\right)$$

Receipts

$$=Fw(ERR) = 10{,}000\left(\frac{F}{P}, ERR, 4\right)$$

$\Rightarrow ERR$=14.10% < MARR $\overset{X}{\Rightarrow}$not desireable

Equal life - Payback period without interest (دوره برگشت بدون بهره)

- **How long does it take to recover only initial costs?**
  - The **time when** the **sum of the net cash flows**, both **receipts and disbursements**, to date is **nonnegative (zero or positive)**.
- ***Weaknesses:***
  - Ignores interest on the initial costs.
  - Ignores salvage value.
  - Ignores the time value of money.
  - Ignores all cash flows that occur after the payback period.
  - Ignores life of the equipment.
- ***Strengths:***
  - Good analytical tool when there is a high degree of uncertainty.
  - Good analytical tool when one wants to recoup the investment as soon as possible.

Equal life - Payback period without interest (دوره برگشت بدون بهره)

How to make decision using payback period without interest?

- Should be **used** in **conjunction** with **other quantitative methods** such as **PW (NPV)**
  - Can be **used to help** select **between two alternatives** with **similar PWs (NPVs))**

*If **payback period without interest** is **longer** than the **life of the alternative**, the **alternative** should be **rejected**.*

Equal life - Payback period with interest (دوره برگشت با بهره)

- **How long does it take to recover initial costs plus interest?**
  - The **point in time** the **present value** of the **net cash flows** (both cash receipts and disbursements) are **equal to or greater** than **zero**.
    - ✓ Includes **interest** on the **capital investment** + **time value of money**
- ***Weaknesses:***
  - Ignores salvage value.
    - ✓ **Unless** the **payback period** is **longer than** or **equal** to the **useful life** of the alternative
  - Ignores all cash flows that occur after the payback period.
  - Ignores life of the equipment.
- ***Strengths:***
  - Good analytical tool when there is a high degree of uncertainty.
  - Good analytical tool when one wants to recoup the investment as soon as possible.

Equal life - Payback period with interest (دوره برگشت با بهره)

How to make decision using payback period without interest?

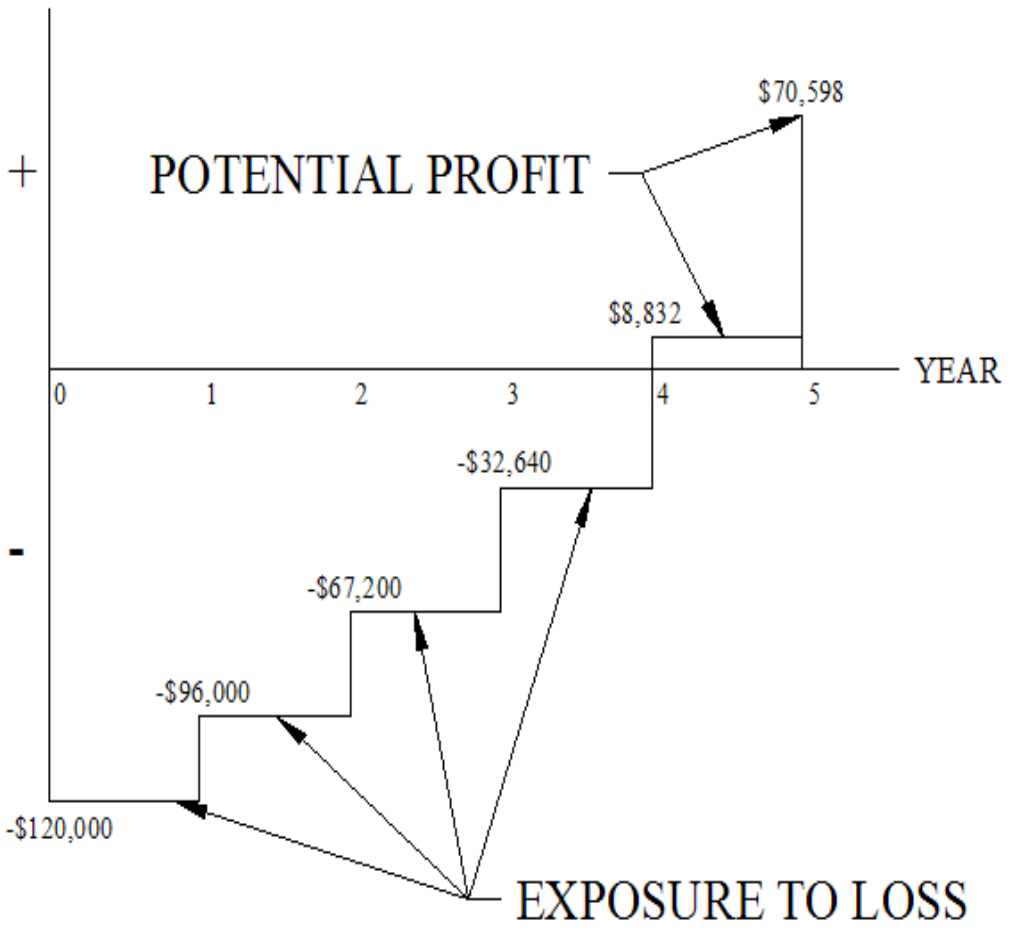
- It is a **better tool** than the **payback period with interest** because it **incorporates interest** on the **investment**.

- Should be **used** in **conjunction** with **other quantitative methods** such as **PW (NPV)**
  - Can be **used to help** select **between two alternatives** with **similar PWs (NPVs))**

*If **payback period with interest** is **longer** than the **life of the alternative**, the **alternative** should be **rejected**.*

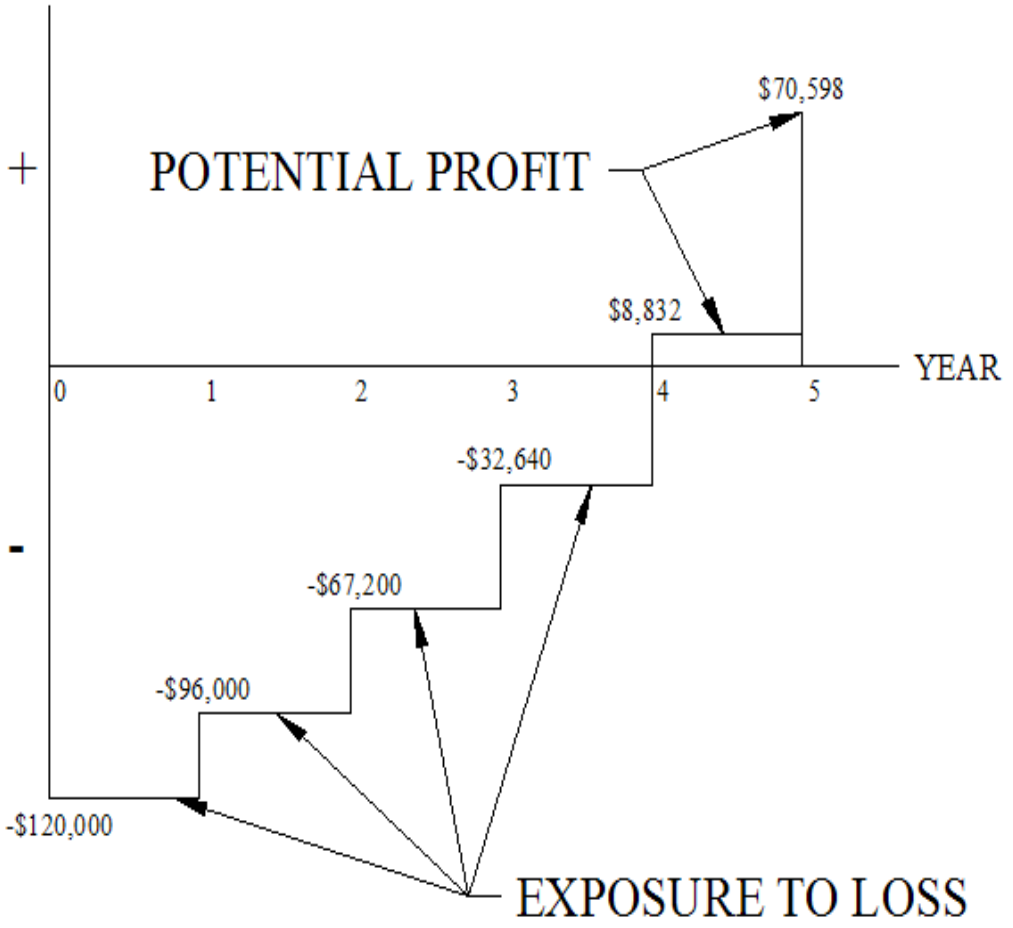
Project balance

- A **graphical method** that shows the **potential profit** or **exposure** to **loss** for **any period** during an **alternative's life** in which the **MARR** is included.
- It is **calculated** by **determining** the **net cash flow to date** for **any alternative**, including **interest** on the **outstanding investment**.



Project balance

- The **below** items will be shown by **project balance**:
  1. Future worth
  2. Payback period with interest
  3. Profit or loss of the years
  4. Time when the project will be profitable.
  5. How much money is under the risk
  6. Best tool when projects have similar PWs (NPVs)



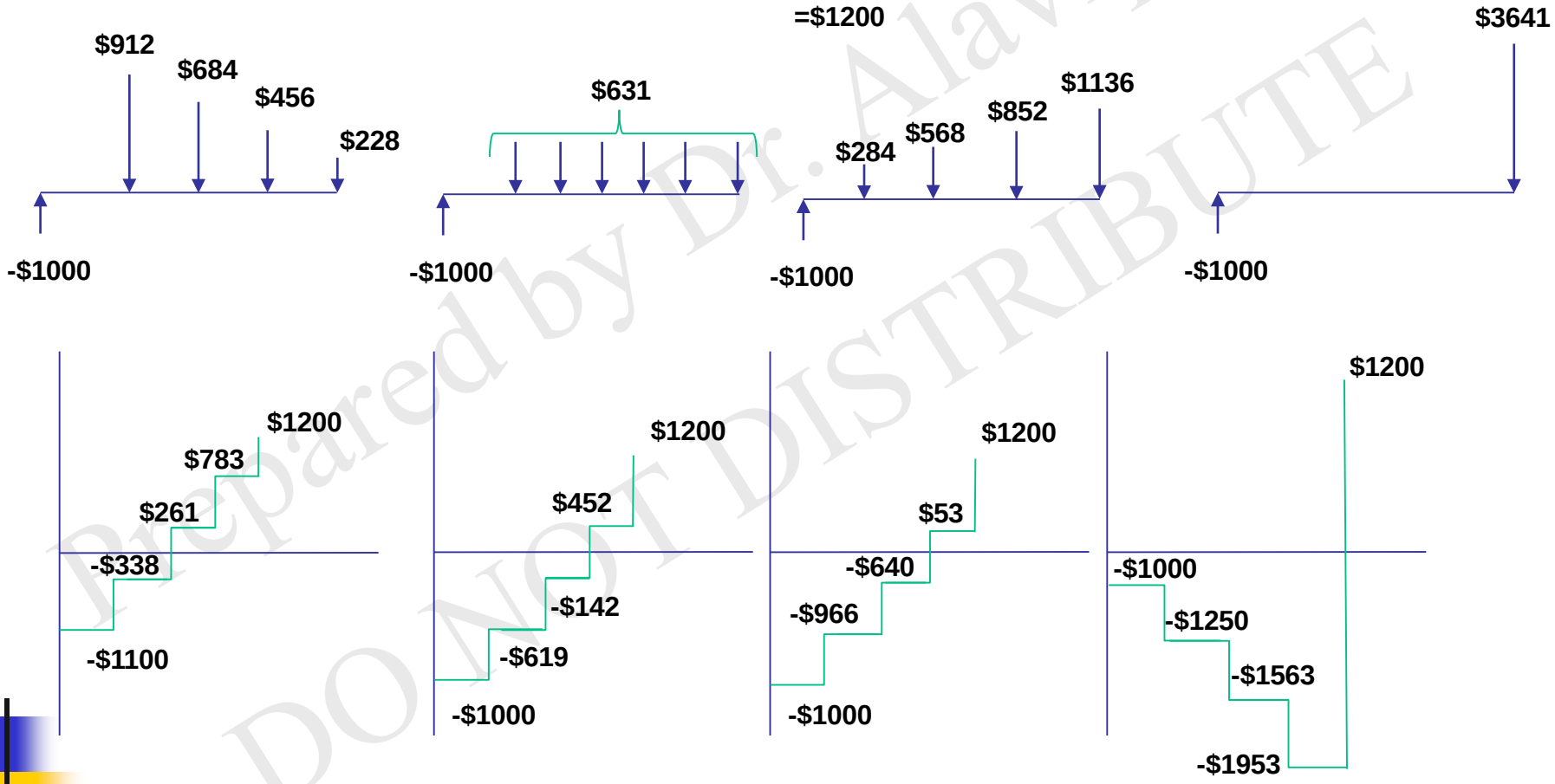
Project balance

- ***Weakness***
  - It **ignores** the **salvage value** until **the end of the study period**.

- ***Strength***
  - It **incorporates** **future worth**—which is **mathematically related** to the **NPV**
  - It **incorporates** **payback period with interest**
  - It **shows** the **potential profit** or **potential loss** for **each of the years**
  - It is **useful** for **risky investment** because it shows the **exposure to loss** for the project.
  - It is also **useful** for **comparing projects** with **similar future worth** (or NPV or AEs) and **similar payback periods with interest**.

Project balance – Example

Exampel: MARR=25% ⇒ *Profit of all alternatives are equal*



Unequal life - Study period

- The **time** which **alternatives** are **compared**.
- Determining the **study period** is difficult as **determining the MARR**. It is selected in one of the following ways:

1. **Company policy**
   - For example, a company may have a **policy** that all **investments** are compared over a **five-year period**.

2. **May be matched to the life of one of the investments**
   - **Common practice** is to **match** the study period to the **alternative with either** the **longest** or **shortest life**

3. **May be set to the least common denominator of the alternative lives**
   - For example, when comparing alternatives that have lives of 3 years, 4 years, and 6 years, the **least common dominator is 12 years**.

4. **May be set based on the length of a need.**
   - For example, if the company **needs to purchase** some **earthmoving equipment** to fulfill its obligation on a highway project with an **expected duration of four years**, the study period could be **set to the life of the project**.

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