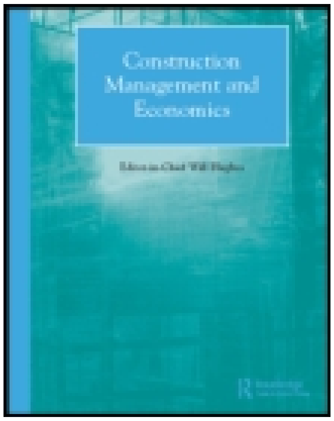

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Heuristic method for multi-project finance-based scheduling

Ashraf Elazouni [a]
[a] King Fahd University of Petroleum & Minerals , Saudi Arabia
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Heuristic method for multi-project finance-based scheduling

ASHRAF ELAZOUNI*

King Fahd University of Petroleum & Minerals, Saudi Arabia

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A heuristic method is proposed for scheduling multiple projects subject to cash constraints. The heuristic determines cash availability during a given period, identifies all possible activities' schedules, determines the cash requirements for each schedule, ranks schedules based on the contribution to minimizing the increase in the project duration, schedules all activities of the selected schedule and determines the impact of the scheduled activities on the project cash flow. The effectiveness of the heuristic method was validated by comparing the results with the optimum results obtained by using the integer programming (IP) technique for 15 networks comprising up to 60 activities. The comparison indicated that the solutions obtained using the proposed heuristic are very comparable to the optimum solutions. An example of two concurrent projects was presented to demonstrate the proposed heuristic method. The proposed heuristic offers the ultimate flexibility to enter cash outflows and inflows at the actual occurrence time, the ability to rationalize the scheduling process, the flexibility to either devise or update schedules, and the ability to schedule practical-size multiple projects. Finally, this heuristic can be easily coded in software to help managers schedule projects under finance-constrained conditions.

Keywords: Cash flow management, construction finance, financial management, heuristic, scheduling.

Introduction

As contractors continue to face intensified competition and low profit margins, the major constraining factor for managing projects and acquiring project resources will probably be the availability of cash to finance new and continuing projects (Smith-Daniels *et al.*, 1996). Thus, a crucial factor for construction contractors to run a sustained business is determined by the ability to procure adequate cash to execute construction operations as scheduled. The procurement of cash, termed as financing, has always been the first concern of contractors. Contractors will definitely breach the obligation to adhere to schedules if adequate funds have not been made available to cover the expenditures of the scheduled activities at any time. This fact emphasizes a pressing need to integrate the scheduling and financing functions of construction project management.

Unfortunately, this integration is entirely missing in the current research and the commercial scheduling software in construction. The most likely reason for the absence of this integration is that people normally consider scheduling and financing as two different concerns of people working on two distinct areas of production planning and business management respectively. Another reason is that the advent of the critical path method (CPM) scheduling technique was initially in an environment which is alien to construction. The vital concept and technique of finance-based scheduling caters for this integration by setting a balance between the activities' disbursements which represent the expenses caused by labour, equipment, materials and subcontractors (cash outflow) and the cash available through owners' payments (cash inflow). Besides owners' payments, contractors often procure additional funds from external sources including banks to supplement owners' payments.

Contractors typically procure cash to partially finance projects by establishing bank credit lines. Under these agreements, bankers often require contractors to deposit owners' progress payments (cash inflows) into the credit-line accounts to continually

*E-mail: elazouni@kfupm.edu.sa

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reduce the outstanding debit. Contractors gain the benefits of this requirement through the reduction achieved in the financing costs. Contractors charge cash outflows against, and deposit cash inflows into, the credit-line accounts as at the ends of periods. Accordingly, the values of the negative cumulative balance as at the ends of periods are determined. The value of the negative cumulative balance at any time constitutes the outstanding debit due to the bank. The financing costs as at the ends of periods are determined by applying the prescribed interest rate to the outstanding debit.

In addition to financing costs, another concern of financing is the amount of cash being made available. Since cash is a scarce resource, it is very unlikely that bankers would not set limits on the credit allocated to the established credit lines. The credit limit specifies the maximum value the negative cumulative balance is allowed to reach during the course of the project. Finance-based scheduling specifies activities' start times such that the contractor's negative cumulative balance at any time never tops the specified credit limit. Thus, finance-based scheduling achieves the desired integration between scheduling and financing by incorporating financing costs into the project total cost as well as scheduling to constrained credit limits. The remaining part of this section outlines the previous research efforts related to the determination of financing costs and the consideration of finance constraints in project scheduling.

Until the 1970s, the financing cost being indicated by the time value of money was entirely omitted in project scheduling (Kazaz and Sepil, 1996). Then, the NPV was first introduced by Russell in 1970 to determine the event times to maximize the NPV with no consideration given to any kind of resource. In an extended effort, Kazaz and Sepil (1996) invalidated the assumption made by Russell that cash inflows occur at the realization times of some events during the course of the project. Kazaz and Sepil developed a mixed IP formulation to maximize the NPV where the cash inflows occur as progress payments for the work completed during each month and cash outflows occur at the completion of activities. However, Kazaz and Sepil did not present any resource-constrained scheduling technique.

Subsequently, the problem of NPV maximization was expanded by other studies (Russell, 1986; Padman *et al.*, 1997) to include resource constraints. These two studies evaluated the performance of heuristic rules in scheduling resource-constrained projects to maximize the NPV of cash flows. In a more recent research effort, Chiu and Tsai (2002) proposed an efficient priority-based heuristic rule for the resource-constrained multi-project scheduling problem to maximize the project NPV. However, the research trends in these research efforts did not present cash-constrained scheduling techniques.

The problem of NPV optimization with capital-constrained scheduling constitutes a sub-problem of the previous resource-constrained scheduling. The study by Doersch and Patterson (1977) was the pioneer work to maximize the NPV with a limit on the amount of capital available followed by two remarkable papers of Smith-Daniels and Smith-Daniels (1987) and Smith-Daniels *et al.* (1996). These models suffered from the major drawback of using the time unit of a month to specify the durations of activities and determine activities' shifts. In construction projects, the working day is the most frequently used time unit to estimate activities' durations and determine activities' shifts. In addition, these models assumed that cash inflows occur at the realization times of some events during the course of the project. Typically, cash inflows in construction projects occur at the ends of fixed periods set forth by the owner in the contract. Accordingly, these capital-constrained scheduling models can accommodate financial planning of a big enterprise implementing a set of small projects each represented by a node of a large network that combines all the small projects; the financing of each small project may yield a requirement and a payback (Doersch and Patterson, 1977).

Likewise, the vast majority of cost-optimization scheduling techniques in construction projects entirely discarded the financing costs. Few notable research efforts in construction have identified financing costs as a project cost component. Hegazy and Ersahin (2001) developed a spreadsheet-based model that combines a CPM network scheduling with time/cost trade-off, resource allocation and levelling, and cash flow management. The financing cost was obtained from the cash flow calculations and added to the total project cost. Li (1996) presented a mathematical model to schedule multiple sub-projects with the objective of minimizing the construction costs. This model considered the financing cost associated with the investments on the individual sub-projects. Attempting to consider the financing costs, Karshenas and Haber (1990) divided the cost of a resource in a construction project into resource mobilization cost and resource use cost. However, Karshenas and Haber considered the use cost of cash, which is supposedly the financing costs, as the sum of activities' costs for a given period. However, Hegazy and Ersahin, Li, and Karshenas and Haber did not consider fund limitations in project scheduling.

Finance-based scheduling

The special terminology of finance-based scheduling was introduced to reflect the merits this concept and technique offer to suit the peculiarity of construction projects, incorporate the financing costs and devise schedules at specified credit limits. The distinctive features of finance-based scheduling are explained below.

Extension scheme

The extension scheme is a special framework for extending the project duration while keeping the networking basics intact. It allows for activities' shifting such that the negative cumulative balance values at the ends of periods never top the specified credit limit. An extension scheme can be obtained for a CPM network by appending the predetermined extension increment to the total floats of the individual activities. The extension scheme provides a well-defined timeframe of definite boundaries to formulate a model and search for a solution.

Unique treatment of cash

A distinctive feature of cash, which makes it a unique resource, is that the produced works in turn yield cash through cash inflows which is obviously not true for the other resources. The typical practice in construction is to utilize the yielded cash at any time to finance the remaining activities of the projects.

Look-back feature

The determination of the cash flow parameters at the end of a certain period builds on the respective transactions of previous periods. This feature (referred to as look-back feature) allows contractors enter a cash inflow at a given period to the cash flow model few periods later when the money is actually collected. This enables the utilization of the collected cash inflows of a project, which constitute a major financing source, to finance the same project during its subsequent periods. This look-back feature accommodates the practice in construction whereby the receipt of cash inflows is often deferred by one or more periods.

Credit-line bank accounts

External financing sources, especially credit-line bank accounts, supplement cash inflows to procure cash since it is only in very rare situations when cash inflows alone secure the required cash. A credit-line account works as a financial pool that processes cash outflows for all contractors' ongoing projects, and receives the amounts of funds procured through cash inflows, bank credit and other external financing sources. Owing to the incorporation of external financing sources, it is very unlikely that cash availability would be even across periods. The bank account determines on a periodical basis the overall liquidity situation of the contractors so they will be able to effectively achieve the best utilization of the available cash to reduce the total financing costs.

Time unit of day

It is to be noted that though cash inflow and outflow transactions have to be determined and processed on a periodic basis, yet the activity shifting in finance-based scheduling is specified using the time unit of a working day. It is almost mandatory in construction to specify the durations of activities and present schedules using the unit of a working day. Using other time units (e.g. a week or month) as has been followed in literature of capital-constraint scheduling is not practical in the scheduling of construction projects.

Employing different scheduling techniques simultaneously

Since scheduling to the available finance is a common objective among all construction projects regardless of their categories, finance-based scheduling offers common grounds to integrate concurrent projects being scheduled using CPM, linear scheduling and line of balance (LOB). The management of multiple projects simultaneously based on the financial aspect while fulfilling the different objectives of individual scheduling techniques is a big stride towards realistic scheduling for construction contractors.

Elazouni and Gab-Allah (2004) devised finance-based schedules for a single project with the objective of time minimization using the IP technique based on CPM networks. However, optimal solutions to the cash-constrained project scheduling problems are impractical if not impossible to generate for large projects since the general class of these problems is NP-complete (Garey and Johnson, 1979). Thus, it becomes more appropriate to resort to heuristics to solve project scheduling problems consisting of hundreds or even thousands of activities. Heuristic methods offer the merit of being simple to understand, easy to apply, very inexpensive to use in computer programs and are able to rationalize the scheduling process and make it manageable for practical-size projects (Hegazy, 1999). Managers have already displayed receptiveness to the use of heuristics that have been incorporated into a variety of commercial project management computer packages (Smith-Daniels and Smith-Daniels, 1987). A heuristic method is proposed herein to implement the concept of finance-based scheduling on multiple concurrent projects.

Heuristic finance-based scheduling

The proposed heuristic method determines the cash available to schedule activities during a given period; identifies all possible schedules of the eligible activities; determines the cash requirements and the impact of each schedule on the total project duration; selects the schedule that is the best available for its fund requirement and causes the minimum increase in the project duration; and updates the calculations of the cash flow as of the end of the current period. The heuristic algorithm proceeds one period at a time till all project activities are scheduled. The steps of the heuristic method are illustrated in Figure 1 and explained below.

Determination of cash availability

The equations in this section are presented conforming to the financial terminology used by Au and Hendrickson (1986). A project cash outflow plus the overheads and taxes during a typical project period t is represented by E_t, the corresponding cash inflow is received at the end of the same period t and is represented by P_t.

The net cumulative balance as of the end of the period t after receiving a cash inflow P_t is N_t where

$$N_t = F_t + P_t \tag{1}$$

$$F_t = N_{t-1} + E_t \tag{2}$$

Accordingly, the total financing cost at the end of period t is I_t, where

$$I_t = rN_{t-1} + r\frac{E_t}{2} \tag{3}$$

The first component of I_t represents the financing cost for period t on the net cumulative balance N_{t-1}; the second component approximates the financing cost on E_t for period t; and r is the financing rate per period.

If the contractor decided to compound the periodical interest I_t at a financing rate r, the cumulative balance at the end of period t including accumulated financing costs is $\hat{F}_t$, where

$$\hat{F}_t = F_t + \hat{I}_t \tag{4}$$

The second term $\hat{I}_t$ represents the accumulated financing costs at the end of period t, where

$$\hat{I}_t = \sum_{l=1}^{t} I_l(1+r)^{t-l} \tag{5}$$

Similar to Equation 4, the net cumulative balance including accumulated financing costs at the end of period t is $\hat{N}_t$, where

$$\hat{N}_t = N_t + \hat{I}_t \tag{6}$$

For period $t-1$, Figure 2 shows the cumulative balance at the end of this period including accumulated financing costs $\hat{F}_{t-1}$, and the net cumulative balance $\hat{N}_{t-1}$. In addition, Figure 2 shows the same parameters as at the end of period t.

The objective of finance-based scheduling is to search for a minimum-duration schedule such that the values of the negative cumulative balance at any period t including the accumulated financing costs $\hat{F}_t$ never top the value of the specified credit limit W. These constraints can be formulated as follows:

$$|\hat{F}_t| \leq |W| \qquad t = 1, 2, 3, \ldots, L \tag{7}$$

The maximum amount of cash available to schedule activities at any period t can be determined when inequality 7 becomes Equation 8 below.

$$\hat{F}_t = W \tag{8}$$

$$W = F_t + \hat{I}_t \tag{9}$$

$$W = (N_{t-1} + E_t) + (\hat{I}_t) \tag{10}$$

If U_t is the total of the cash outflow during period t of the leftovers of the activities scheduled before plus the overhead costs for period t and tax applied to the total; V_t is the maximum amount of cash outflows of activities yet to be scheduled during period t with tax applied; thus, the summation of U_t and V_t equals E_t as defined above.

$$E_t = U_t + V_t \tag{11}$$

The accumulated financing costs $\hat{I}_t$ at the end of period t are equal to the accumulated financing costs $\hat{I}_{t-1}$ at the end of the previous period $t-1$ compounded for one period plus the financing cost I_t for period t. Accordingly, Equation 10 can be rewritten as:

$$W = (N_{t-1} + U_t + V_t) + (\hat{I}_{t-1}(1+r) + I_t) \tag{12}$$

Substitute for I_t as in Equation 3:

$$\begin{aligned} W = {} & N_{t-1} + U_t + V_t + \hat{I}_{t-1} \\ & + \hat{I}_{t-1}(r) + N_{t-1}(r) + (U_t + V_t)(r/2) \end{aligned} \tag{13}$$

$$\begin{aligned} W = {} & N_{t-1} + N_{t-1}(r) + \hat{I}_{t-1} + \hat{I}_{t-1}(r) \\ & + U_t(1 + r/2) + V_t(1 + r/2) \end{aligned} \tag{14}$$

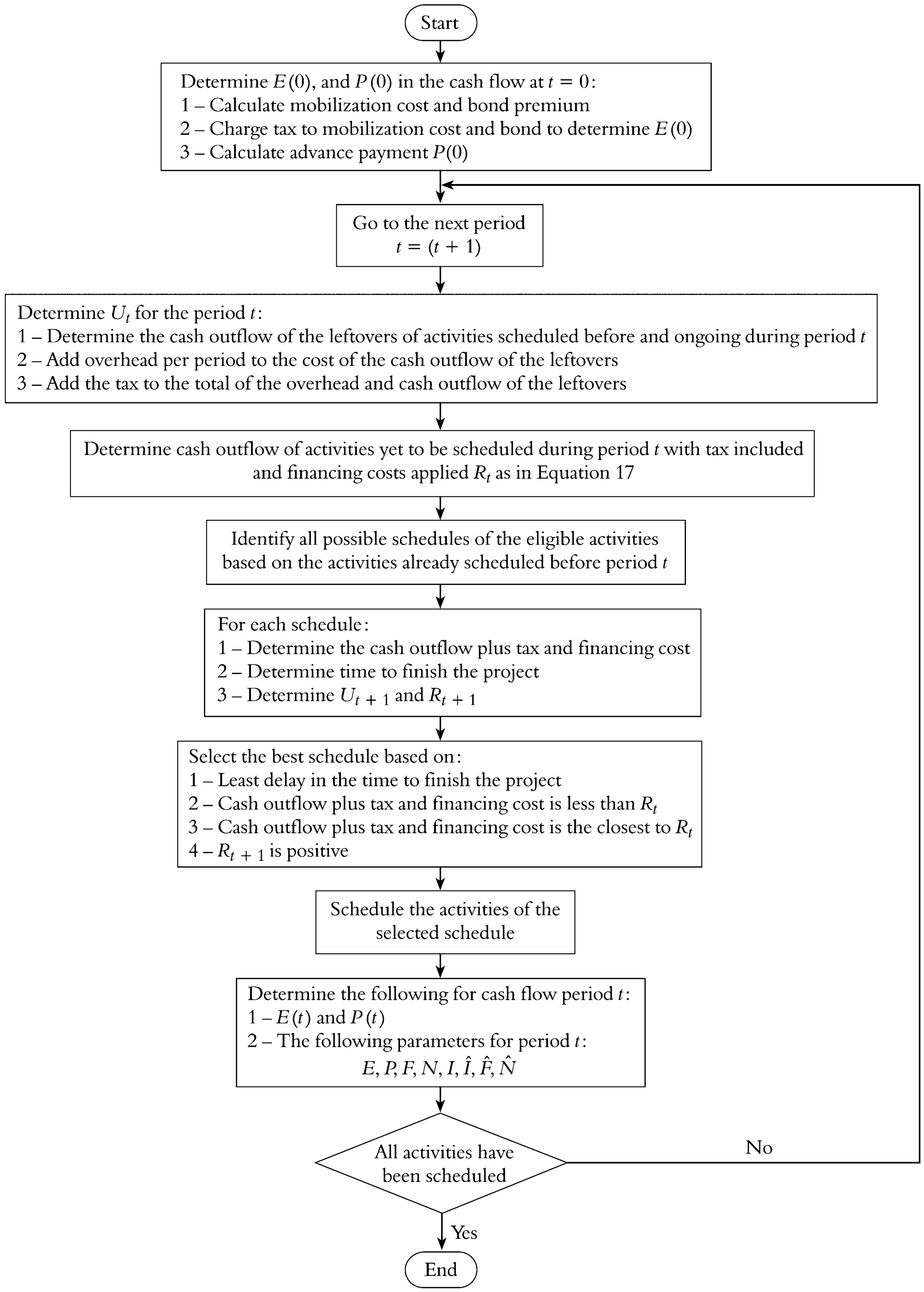

Figure 1 Flowchart of the heuristic algorithm

$$V_t(1+r/2) = W - N_{t-1} - N_{t-1}(r) - \hat{I}_{t-1} - \hat{I}_{t-1}(r) - U_t(1+r/2) \quad (15)$$

$$V_t(1+r/2) = W - \left(N_{t-1} + \hat{I}_{t-1}\right) - N_{t-1}(r) - \hat{I}_{t-1}(r) - U_t(1+r/2) \quad (16)$$

If R_t is the V_t with the financing cost rate applied for one period t.

$$R_t = W - \hat{N}_{t-1} - N_{t-1}(r) - \hat{I}_{t-1}(r) - U_t(1+r/2) \quad (17)$$

Equation 17 indicates that the maximum amounts of cash outflows of activities yet to be scheduled during

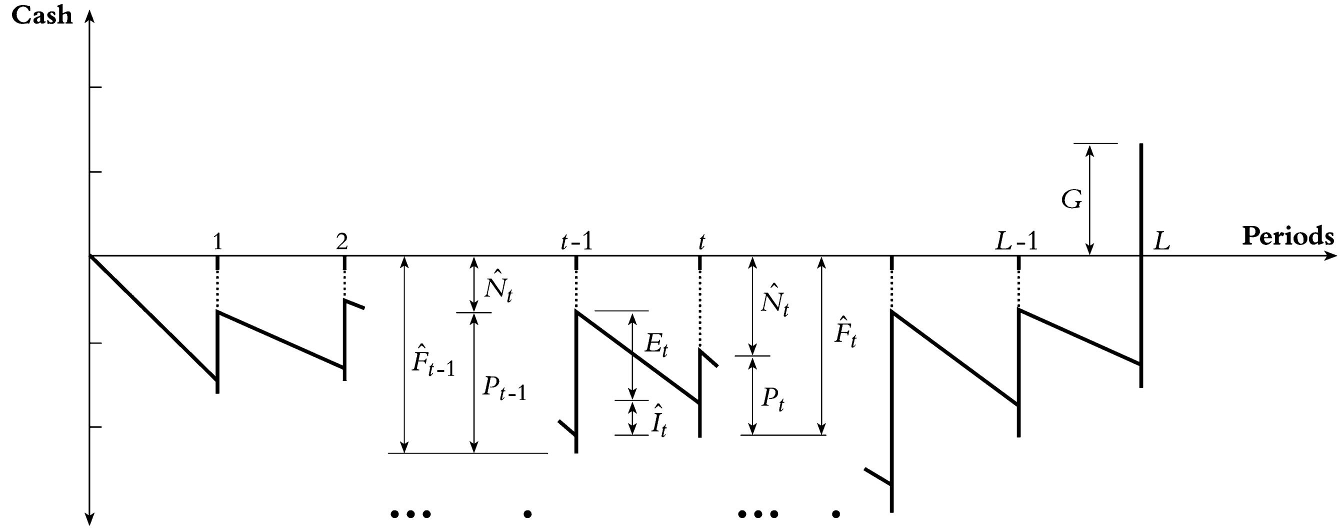

Figure 2 Cash flow profile for a typical project

period t with tax included and financing costs applied should not top the values of R_t so the values of the negative cumulative balance never top the credit limit of W.

Identification of possible activities' schedule

Generally, the cash flow of the project changes at discrete points in time when the cash inflows are deposited into the credit-line account. These discrete points in time determine the cash inflow periods of project cash flow. Thus, it is compulsory that contractors follow the same cash inflow periods for the purposes of cash outflow calculations in particular and for the cash flow analysis in general. This implies that scheduling is logical to proceed such that sets of eligible activities are considered for possible scheduling during the cash inflow periods which cash inflows take place at their starts.

Typically, the cash inflows occur regularly at the ends of fixed periods stipulated in the contract. Occasionally, contracts stipulate that cash inflows occur upon a predetermined progress in terms of money reached which results in irregular occurrence of cash inflows. In this case, the periods associated with these predetermined progress milestones need to be determined as scheduling process proceeds and the cash inflows are entered to the cash flow at these periods.

The initial schedule of a set of activities during a certain period can be identified by considering all activities eligible to start at the earliest times upon the completion of the respective preceding activities. Additional schedules emerge when a certain activity crosses the border of its current period by at least one day. These activities' crossings change the cash outflow during the current period and consequently increase the possibility to meet constraints in fund. The total number of schedules can be determined by enumerating all possible activities' crossings. For all the determined schedules, the start times of activities are plugged into the CPM network devised up to the current period and the network computations are completed to determine the impact of each schedule on the project duration.

Selection of the best schedule

The heuristic algorithm works on a periodical basis to schedule activities fulfilling the financial constraint of credit limit, and constructs the cash flow as the scheduling process proceeds. In addition, the algorithm achieves the maximum utilization of the fund available during a certain period. The objective is to schedule the project under the specified credit limit while minimizing the project duration. At the beginning of the project, as shown in the flowchart in Figure 1, E_0 encompasses the mobilization costs and bond premium plus tax, and P_0 constitutes the advance payment. For any subsequent period t, U_t and R_t are determined as explained earlier.

The possible schedules of activities during the period t are identified along with the activities' cash outflows plus tax, and financing costs; the time to finish the project; and the values of U_{t+1} and R_{t+1}. The best schedule is selected which exhibits the minimum delay in the project completion; the summation of the cash outflow plus tax and financing costs is less than R_t; the summation is the closest to R_t; and the value of R_{t+1} is positive to ensure that cash will be available to implement the leftovers of this schedule during the subsequent period t+1. Finally, E_t and P_t are determined for period t and consequently the other financial parameters of the cash flow.

Heuristic validation

The proposed heuristic algorithm was validated by comparing its results with the exact results obtained by using the IP technique based on five CPM networks of 12, 24, 36, 48 and 60 activities. For the purpose of validation, the results constitute the project durations obtained by the IP technique and the heuristic algorithm. Figure 3 shows the CPM network of a project consisting of 12 activities with four activities (A, B, C and D) being implemented over three sections (a, b and c). The 24, 36, 48 and 60-activity projects repeat the same four activities of the 12-activity project for 2, 3, 4 and 5 times respectively. For instance, the 24-activity project repeats the same four activities A, B, C and D over three more identical sections (d, e and f). For example, activities Ba, Bb and Bc are duplicated to activities Bd, Be and Bf with the same respective durations such that Bd depends on Bc and Ad; Be depends on Bd and Ae; and Bf depends on Be and Af.

The overhead costs of each project were considered as $3500 per week. The contract is a cost-plus fee with a mark-up of 20% applied to the total weekly summation of cash outflows and the overheads. The owner's progress payments (cash inflows) are obtained one week after the submission of the weekly pay requests, with no advance payment considered. A financing rate of 1.5% per week was considered. The retention percentage of each cash inflow amounts to 16% and the retained money is to be paid eight weeks after the completion of the projects.

IP models were formulated for the 12, 24, 36, 48 and 60-activity networks based on extension schemes of 10-day extension increments as in Elazouni and Gab-Allah (2004). The models formulate objective functions that minimize the project duration under the financial, sequence and activities' shifting constraints. A program was developed in an earlier study (Elazouni and Hassan, 2008), which uses C# to formulate the constraints of the model and allows the user to save in text files. The models were solved using the optimization software of Lingo 10 (2006). Three credit limits were chosen that produce increases in total durations falling in the three ranges of 0–3, 4–6 and 7–10 days. Table 1 indicates clearly that the results of the heuristic are very comparable to the results of the IP. The difference in the total duration amounted to only one day less in favour of the IP in only three cases out of the 15 cases.

However, optimal solutions to the cash-constrained project scheduling problems are impractical if not impossible to obtain for large projects since the general class of these problems is NP-complete (Garey and Johnson, 1979). It took a laptop of processor 1.86GHz

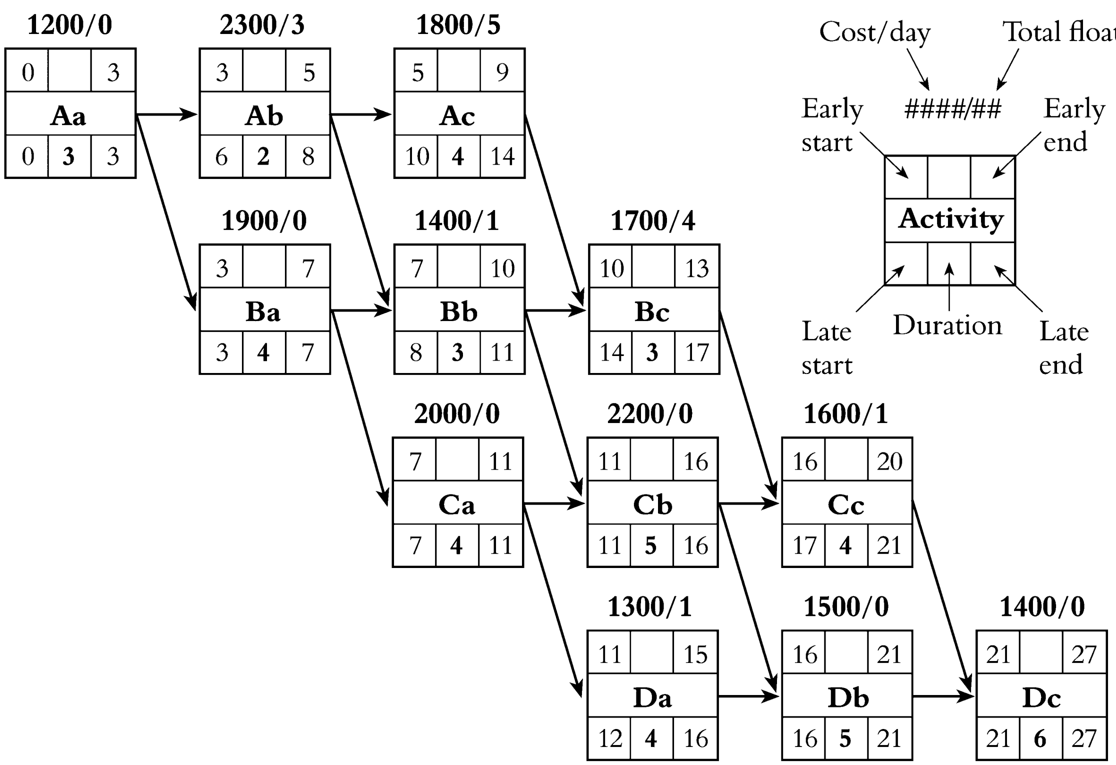

Figure 3 Critical path method network of 12-activity project

and 1.00GB of Ram up to 16 hours to solve the model for the network of 60 activities. When the project size was increased to 72 activities, no solution was encountered by Lingo after a run time of a few days. Since project scheduling problems often consist of hundreds or even thousands of activities, it becomes more appropriate to resort to heuristics to solve medium and large-scale project scheduling problems.

The IP exhausts the entire solution space searching for a global optimum solution of the whole network. On the other hand, the heuristic divides the whole domain of the problem into sub-domains, exhaustively enumerates on a weekly basis the possible schedules, and finds the best schedule for each individual week based on the criteria discussed earlier. The sub-schedules for the individual weeks fulfil the sequence and credit limit constraints. However, the collective schedule obtained by separately devising sub-schedules does not guarantee a globally optimized schedule as was evidenced by the results discussed above.

The IP technique is a static model that cannot adequately model all the types of cash outflows and inflows and simultaneously perform the necessary adjustments during the searching process. For instance, a 10-day extension scheme of the 12-activity network above is of 37 days (equivalent to eight weeks). The IP model formulates the payment of the retained money as part of the last payment which occurs at the end of the last week. Had the credit limit been relaxed and the model found a solution at a project duration of 29 days (equivalent to six weeks), the payment of the retained money would not be included in the last payment, which would lead to erroneous calculation of the profit at the end of week 7. Another problem will arise in the same case if the contract stipulates that the advance payment will be repaid on a weekly basis for eight weeks. The later problem could be more serious as this will impact on the accuracy of the negative cumulative balance at the end of weeks 1 to 5, which might produce erroneous results. In contrast, this shortcoming does not exist in heuristic method since it does not entail a prior formulation of the whole problem and adjustments can be made at any time.

Application of the heuristic

The heuristic algorithm was demonstrated using two concurrent projects of the 12-activity network shown in Figure 3. The two 12-activity projects were set up such that the start of the second project lags four weeks behind the start of the first project. The duration of each project spans over 27 working days which makes six weeks, and the duration of the two projects together spans over 47 days which makes 10 weeks. The financial data and the contractual terms of the two projects are presented in Table 2. Table 3 shows for the first project the activities' cash outflow daily rates with the overheads, mobilization costs, tax, mark-up and bonds calculated pro-rata to determine the activities' prices (cash inflows) on a daily basis.

Three schedules were constructed at credit limits of \$53 000, \$50 000 and \$47 000. At each credit limit, two schedules were devised with priorities of 1–0, and 0–1 assigned to the two projects. A priority of 'one' assigned

Table 1 The total duration and the maximum negative cumulative balance of the original network, and the total duration of heuristic and integer programming

Network	Original network duration (days)	Original maximum negative cumulative balance \$	Credit limits \$	Total network duration (days)	
				Heuristic method	Integer programming
12 activities	27	50 272.90	40 000	29	29
			37 000	32	32
			32 000	35	35
24 activities	42	70 394.90	55 000	43	43
			50 000	46	46
			43 000	52	52
36 activities	57	75 238.30	58 000	59	58
			54 000	62	62
			50 000	66	66
48 activities	72	77 751.60	58 000	75	75
			56 000	78	77
			53 000	82	82
60 activities	87	79 417.23	63 000	88	87
			58 000	93	93
			57 000	94	94

Table 2 The financial data and the contractual terms of the two projects

Category	Item	First project	Second project
Interest rate (%)	Interest rate per week	0.3	0.3
Financial data	Original duration (days)	27	27
	Original duration (weeks)	6	6
	Overheads per week ($)	3500	2500
	Mobilization costs ($)	20 000	15 000
	Tax percentage	2	2
	Mark-up percentage	20	20
	Bond premium ($)	2200	1300
Contract terms	Advance payment percentage of bid price	6	4
	Weeks to retrieve advance payment	6	6
	Retained percentage of pay requests	5	7
	Lag to pay retained money after last payment (weeks)	0	0
	Weeks to submit pay requests regularly	1	1
	Lag to pay payment requests (weeks)	1	1

to a particular project indicates that in each week, the total available fund will be allocated to select the best schedule of this project and the remaining fund, if any, will be allocated to select the best schedule of the second project. Table 4 presents the total duration of the individual projects and the two projects together, the profit as of the end of the two projects, and the maximum negative cumulative balance values. The results in Table 4 indicate that the profit values at the credit limit of $53 000 were slightly less than the profit value of the original schedule though the completion time of the individual projects and the two projects together were identical in terms of the number of periods. This slight difference can entirely be attributed to the difference in the financing cost since the overhead costs were the same. At credit limits of $50 000 and $47 000 the profit values decreased drastically owing to the increase in overheads. Table 5 presents the cash flow parameters throughout the 12-week duration of the schedule that corresponds to the credit limit of $50 000 with priority being assigned to the activities of the first project.

Figure 4 shows the scheduling of the activities during the fourth week at a credit limit of $53 000 with priority assigned to allocate cash to the first project. The situation before scheduling activities at the beginning of the fourth week was such that activities Aa, Ab, Ba, Ac, Bb, Ca and Da were completed; activity Cb was started during the third week but still has two remaining days during the fourth week; and activities Bc, Cc, Db and Dc did not start. The set of eligible activities to start during the fourth week encompasses activities Bc, Cc

Table 3 The rates of the cash outflows and inflows of the activities of the first project

Activity	Duration in days	Cost per day $	Total cost $	Total price $	Price per day $
Aa	3	1200	3600	6799.6	2266.5
Ab	2	2300	4600	8688.3	4344.2
Ac	4	1800	7200	13 599.2	3399.8
Ba	4	1900	7600	14 354.7	3588.7
Bb	3	1400	4200	7932.8	2644.3
Bc	3	1700	5100	9632.7	3210.9
Ca	4	2000	8000	15 110.2	3777.5
Cb	5	2200	11 000	20 776.5	4155.3
Cc	4	1600	6400	12 088.1	3022.0
Da	4	1300	5200	9821.6	2455.4
Db	5	1500	7500	14 165.8	2833.2
Dc	6	1400	8400	15 865.7	2644.3

Note: The prices in this table do not include the financing cost. Total cash outflow $ 78 800; overheads $ 21 000; mobilization costs $ 20 000; cash outflow, overheads and mobilization $ 119 800; taxes $ 2396; taxes, cash outflow, overheads and mobilization $ 122 196; mark-up $ 24 439; mark-up, taxes, cash outflow, overheads and mobilization $ 146 635; bond premium $ 2200; total bid price $ 148 835; factor to determine price based on cash outflow (14 8835.2 ÷ 78 800) = 1.888772.

Table 4 The project end times, the total profit, and the maximum negative cumulative balance for the two 12-activity projects

Data	Initial schedule	Credit limit of $53 000		Credit limit of $50 000		Credit limit of $47 000	
		1–0	0–1	1–0	0–1	1–0	0–1
Total duration of the first project (days)	27	29	29	31	46	35	69
Total duration of the second project (days)	27	27	27	31	29	38	39
Total duration of the two projects (days)	47	47	47	51	49	58	69
Total profit at the end of the two projects ($)	45 777.4	45 726.8	45 726.8	39 532.8	31 274.8	36 863.4	11 449.8
Maximum negative cumulative balance ($)	−60 120.6	−52 956.6	−52 956.6	−49 992.2	−49 992.2	−46 940.1	−46 940.1

and Db. The initial schedule of these three activities indicates the early starts of Bc, Cc and Db are at days 15, 18 and 17 respectively. Table 6 presents the total 36 possible schedules of the starts of the three activities. For each schedule, Table 6 presents the total project duration, the cash outflow of each schedule during the fourth week including tax and financing costs, and the value of R_5. Given that R_4 value amounted to $11 756.3, the best schedule is the second in the list. So, activities Bc, Cc and Db are scheduled to start at days 15, 18 and 18 respectively as shown in Figure 4. The remaining fund of $213.4 will not be utilized as the second project is supposed to start the fifth week. Based on the finalized schedule of the fourth week, the cash flow parameters are calculated and consequently the value of R_5. Being of positive value, R_5 indicates that the leftovers of activities Cc and Db can be achieved during the fifth week and an additional fund of $15 162.4 is available to schedule more activities.

Conclusions

A heuristic algorithm was presented to schedule multiple projects under cash constraints. This method identifies all possible activities' schedules, ranks all the identified schedules, selects the best schedule based on the contribution to minimizing the increase in the project duration, schedules all activities of the selected schedule, and determines the impact of the scheduled activities on the cash flow in terms of cash outflows and inflows. The effectiveness of the heuristic method was established by comparing its results with the results obtained by the exact solution obtained by the IP technique. An example of two concurrent projects was presented to demonstrate the heuristic method.

The presented heuristic method is applicable in initial planning as well as control conditions. During project control, updating schedules can be achieved following exactly the same procedure as the original schedule. At the beginning of a given period, the heuristic estimates the remaining durations of the activities in progress. Consequently, the heuristic identifies the eligible activities, and determines the possible combinations thereof. The available cash can be determined more accurately in control conditions based on the actually compiled financial data of the previous part of the project. Finally, the heuristic selects the best combination during the current period.

The extensive computational time the heuristic algorithm takes to enumerate all possible schedules of a set of eligible activities during a given period constitutes a major drawback. This computational time grows exponentially as the number of eligible activities increases and the time span of the period increases. However, this drawback can be alleviated by scheduling only one project of the concurrent projects at a time. This can be achieved either by ranking projects based on the priority for scheduling or by distributing the available fund among the concurrent projects and scheduling projects separately. Fortunately, these two options are practically acceptable. However, this drawback will exist when the set of eligible activities for one project is big or the time span of the period is long.

Devising scheduling in parallel with the simultaneous calculation of the financial parameters on a periodic basis offers high flexibility to enter activities' cash outflows and inflows at the actual occurrence time. This flexibility is in great demand in real construction projects. For instance, if an activity cash outflow occurs in advance or after the completion of an activity, the model allows deduction of the cash outflow amount from the available cash of the appropriate periods before scheduling any activities during these periods. Similarly, if an activity cash inflow occurs in

Table 5 The cash flow parameters throughout the 12-week duration of the schedule of $50 000 with priority assigned to the first project

Financial parameter	Weeks												
	0	1	2	3	4	5	6	7	8	9	10	11	12
Disbursements E ($)	−22644	−15810	−20094	−20094	−27159.2	−25908	−21420	−31416	−22542	−16728	−9690	−3978	0
Payment P ($)	8930.1	0.0	20043.6	27579.8	27579.8	20223.1	31285.1	22871.8	46193.9	30209.4	21163.3	10212.8	11634.8
Cumulative balance F ($)	−22644	−29523.9	−49617.9	−49668.2	−49247.6	−47575.7	−48772.6	−48903.5	−48573.8	−19107.9	1411.525	18596.81	28809.57
Net balance N ($)	−13713.9	−29523.9	−29574.2	−22088.4	−21667.7	−27352.6	−17487.5	−26031.8	−2379.9	11101.5	22574.8	28809.6	40444.4
Financing costs I ($)	−20.5708	−64.8567	−118.713	−118.864	−107.004	−103.865	−114.188	−99.5866	−111.908	−32.2316	0	0	0
Accumulated financing costs $\hat{I}$ ($)	−20.5708	−85.4892	−204.458	−323.935	−431.911	−537.072	−652.871	−754.416	−868.588	−903.425	−906.136	−908.854	−911.581
Cumulative balance with interest costs $\hat{F}$ ($)	−22664.6	−29609.4	−49822.3	−49992.2	−49679.5	−48112.8	−49425.5	−49658	−49442.4	−20011.3	505.3894	17687.96	27897.99
Net balance with interest costs $\hat{N}$ ($)	−13734.5	−29609.4	−29778.7	−22412.3	−22099.6	−27889.7	−18140.4	−26786.2	−3248.4	10198.1	21668.7	27900.7	39532.8

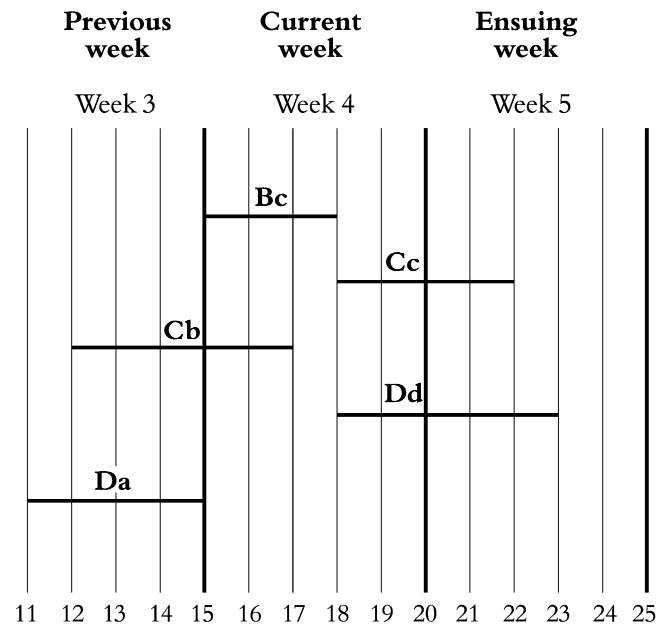

Figure 4 Scheduling of the activities during the fourth week at a credit limit of $53 000 with priority assigned to allocate cash to the first project

advance, the model allows adding the cash inflow amount to the available cash of the appropriate periods before scheduling any activities during these periods.

This heuristic expands the finance-based scheduling technique to deal with multiple projects to determine the activities' start times such that the financial constraint is fulfilled. As a matter of fact, financing should be looked upon as a business concern rather than a project concern. For a profitable and well-sustained business, contractors concern themselves with the timely procurement of funds for ongoing projects altogether. Finance-based scheduling in this context ensures that negative cash flows of individual projects do not add up and surplus cash is properly utilized to schedule activities. Following this concept in scheduling, all concurrent projects can be related to the overall liquidity situation of contractors.

Finally, finance-based scheduling invalidates the underlying assumptions of capital-constrained scheduling that the month is an appropriate time unit to specify the durations and activities' shifts, and the cash inflows occur at the realization times of certain events. In addition, finance-based scheduling offers the dual merits of incorporating financing costs into the calculations of the total project cost as well as devising schedules that are executable at specified credit limits. The heuristic methods offer the merit of ease in expressing any cash outflow and inflow

Table 6 Possible schedules of the first project during the fourth week at a credit limit of $53 000 with priority assigned to the first project

Schedule serial number	Start times			Total duration (days)	Cash outflow plus tax and financing cost ($)	Cash available in fifth period R_5 ($)
	Bc	Cc	Db			
1	15	18	17	28	13 075.20	15 157.50
2	15	18	18	29	11 542.95	15 162.00
3	15	18	19	30	10 010.70	15 166.70
4	15	18	20	31	8478.45	22 832.70
5	15	19	17	29	11 440.80	15 162.40
6	15	19	18	29	9908.55	15 167.00
7	15	19	19	30	8376.30	15 171.60
8	15	19	20	31	6844.05	22 837.60
9	15	20	17	30	9806.40	21 705.10
10	15	20	18	30	8274.15	21 709.70
11	15	20	19	30	6741.90	21 714.30
12	15	20	20	31	5209.65	29 380.30
13	16	19	17	29	11 440.80	15 162.40
14	16	19	18	29	9908.55	15 167.00
15	16	19	19	30	8376.30	15 171.60
16	16	19	20	31	6844.05	22 837.60
17	16	20	17	30	9806.40	21 705.10
18	16	20	18	30	8274.15	21 709.70
19	16	20	19	30	6741.90	21 714.30
20	16	20	20	31	5209.65	29 380.30
21	17	20	17	30	9806.40	21 705.10
22	17	20	18	30	8274.15	21 709.70
23	17	20	19	30	6741.90	21 714.30
24	17	20	20	31	5209.65	29 380.30
25	18	21	17	31	8069.85	21 710.30
26	18	21	18	31	8069.85	21 714.90
27	18	21	19	31	8069.85	21 719.50
28	18	21	20	31	8069.85	29 385.50
29	19	22	17	32	6333.30	21 715.50
30	19	22	18	32	6333.30	21 720.10
31	19	22	19	32	6333.30	21 724.70
32	19	22	20	32	6333.30	29 390.80
33	20	23	17	33	4596.75	26 930.50
34	20	23	18	33	4596.75	26 935.10
35	20	23	19	33	4596.75	26 939.70
36	20	23	20	33	4596.75	34 605.80

methods, the ability to rationalize the scheduling process, and the ability to schedule practical-size networks.

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