Finance-Based Scheduling of Construction Projects Using Integer Programming

Ashraf M. Elazouni[1] and Ahmed A. Gab-Allah[2]

Abstract: Construction scheduling is the process of devising schemes for sequencing activities. A realistic schedule fulfills the real concerns of users, thus minimizing the chances of schedule failure. The minimization of total project duration has been the concept underlying critical-path method/program evaluation and review technique (CPM/PERT) schedules. Subsequently, techniques including resource management and time-cost trade-off analysis were developed to customize CPM/PERT schedules in order to fulfill users' concerns regarding project resources, cost, and time. However, financing construction activities throughout the course of the project is another crucial concern that must be properly treated otherwise, nonrealistic schedules are to be anticipated. Unless contractors manage to procure adequate cash to keep construction work running according to schedule, the pace of work will definitely be relaxed. Therefore, always keeping scheduled activities in balance with available cash is a potential contribution to producing realistic schedules. This paper introduces an integer-programming finance-based scheduling method to produce financially feasible schedules that balance the financing requirements of activities at any period with the cash available during that same period. The proposed method offers twofold benefits of minimizing total project duration and fulfilling finance availability constraints.

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Introduction

A crucial factor for contractors to run a profitable business is the ability to procure adequate cash quickly and to execute construction operations with minimum financing costs. During any period of a project, contractors will never carry out any work that the available cash cannot afford despite the obligation to adhere to schedules. This explicit principle of operation makes the establishment of a balance between the work disbursements and available cash, along the project's duration a very vital concept to producing realistic schedules.

The establishment of bank overdrafts has been one of the prevalent methods of financing construction projects (Ahuja 1976). In practice, contractors establish bank overdrafts with specified credit limits to finance projects and then proceed with project activities so that the disbursements during any period (week or month) will never cause the negative cash flow to exceed the credit limit of the established overdraft. An additional component of financing during a period represents a receipt collected as a reimbursement for work performed during a previous period of the project. Contractors usually deposit these components in the overdraft account to avoid having the negative cash flow reach the credit limit and also to reduce the financing cost. Accordingly, contractors are always interested in developing schedules that prevent project negative cash flow from exceeding the specified credit limit. Since this concept of scheduling is based on finance availability, it is referred to as finance-based scheduling.

There are many techniques that were developed in the literature to schedule construction projects based on different concepts. Critical-path method/program evaluation and review technique (CPM/PERT) scheduling techniques were developed with an underlying concept of minimizing total project duration. Subsequently, many algorithms have been developed to enhance the usefulness of CPM/PERT schedules. Resource management algorithms are among the techniques that offered a substantial enhancement. The literature includes many heuristic, optimal, and suboptimal methods that have been developed to modify CPM/PERT schedules to account for practical considerations of project resources. Despite the fact that money is one of the construction resources, none of the resource management techniques considered the modification of CPM/PERT schedules to balance project expenditures with the available cash for the project's duration.

Generally, traditional resource allocation models that treat available cash as a constrained resource cannot be considered a substitute for the proposed finance-based scheduling method. This is because part of the available cash during a given period is generated as a reimbursement for the work performed during a previous period of the project. Thus a complete profile of available cash for the project's duration, which is a required input to these models, cannot be readily established.

Time-cost trade-off analysis is concerned with selecting appropriate resources including crew size, equipment, methods, and technologies to complete the activities of a project within required durations at minimum cost (Li et al. 1999). Many studies have been conducted on developing mathematical models for solving

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time-cost trade-off problems (TCTP). Rada and Carr (1989) used mixed-integer programming to solve the TCTPs. Liu et al. (1995) used a linear/integer programming hybrid method to perform time-cost trade-off analysis; time-cost problems including both linear and discrete relationships were solved by using mixed-integer programming (Meyer and Shafer 1963; Patterson and Huber 1974); Karaa and Nasr (1986) developed a mixed-integer/linear programming model to minimize the cost of leasing additional resources in construction; Burns et al. (1996) developed a hybrid approach that used linear programming to find a lower bound for the trade-off curve and integer programming to find the exact solution; dynamic programming was used to solve TCTP for networks that can be decomposed to pure series or parallel subnetworks (Robinson 1975; Elmagraby 1993; De et al. 1995); and recently a genetic algorithm was used to solve TCTP (Li and Love 1997; Feng et al. 1997; Li et al. 1999). However, TCTP studies in the literature did not consider the aspect of scheduling activities based on cash availability.

On the other hand of CPM/PERT, schedules, resource-oriented scheduling techniques, including line of balance and the linear scheduling method, include the underlying concept of efficient use of resources. Operating under this concept necessitates the practice of some sort of central control over resources. As a matter of fact, central control over resources might exist in military or job shop scheduling in a factory under single ownership or a single objective, but does not exist in construction projects (Birrell 1980); instead, contractors and subcontractors allocate resources based mainly on the availability of cash. Contractors determine the pace of work considering the efficient use of resources, provided that the cash availability constraint is fulfilled.

Thus none of the present scheduling techniques include the underlying concept or algorithm that establishes a balance between work disbursement and available cash. Consequently, the available scheduling techniques produce financially nonrealistic schedules. This paper presents a method for developing financially feasible schedules by simultaneously relating the scheduled work during given periods to the amount of cash available in the respective periods. The proposed method definitely modifies CPM schedules such that the negative cash flow always undervalues the specified credit limit.

Finance-Based Scheduling

Establishing bank overdrafts has been one of the prevalent methods of financing construction projects (Ahuja 1976). Finance-based scheduling enables producing schedules that correspond to overdrafts of desired credit limits. Control of the credit limit of an overdraft provides many benefits including negotiating lower interest rates with bankers, setting favorable terms of repayment, and reducing penalties incurred for any unused portions of overdraft cash. In addition, the ability to adjust credit limits helps contractors avoid the phenomenon of progressive cash deficit. This situation occurs when cash available in a given month does not allow the scheduling of much work. During the next month, when a reimbursement is expected, the generated income allows scheduling less work and so forth.

In the case of a contractor implementing many projects simultaneously, the use of a finance-based scheduling method is important. This method ensures that negative cash flows for projects do not add up, and, moreover, that surplus cash is used to schedule additional work. Following this concept in scheduling, all existing and planned projects can be related to the overall liquidity situation of the contractor. Consequently, finance-based scheduling helps managers make key decisions regarding bidding for new projects, determining construction rates, and specifying optimum commencement dates of new projects.

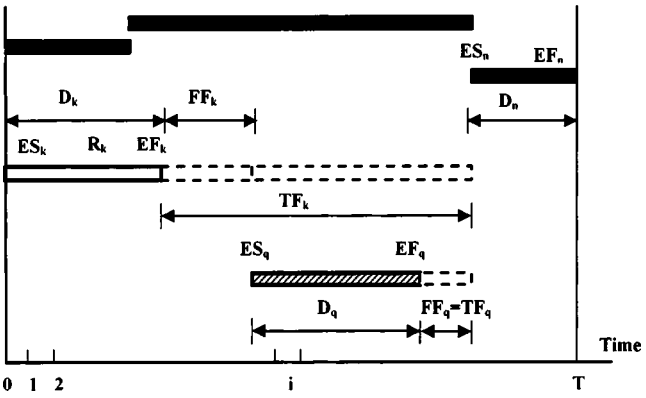

Fig. 1. Initial scheme for CPM schedule with general activity *k*

Moreover, owners are usually interested in determining their capacity to fulfill financial commitments to contractors. When bids are awarded, owners can ask contractors to estimate the progress of payment requirements for the project duration. Owners can then ask for a schedule that produces the desired progress payment requirements-scheme that benefits financial interests.

Finance-Based Scheduling Method

This study proposes a method for devising CPM schedules so that projects can be adequately financed using bank overdrafts of predetermined credit limits, thus making CPM schedules financially executable. This method involves devising an initial scheme for a project, devising an extension scheme, formulating an integer programming model, and searching for a solution. The following sections describe the four phases of the method.

Devising Initial Scheme

The initial scheme for a CPM schedule, as shown in Fig. 1, is basically a bar chart that portrays project activities against time. Moreover, the total float is appended before each activity. The total float represents the maximum shift in an activity without affecting project duration. Fig. 1 shows a general activity k that includes duration D_k, free-float FF_k, total float TF_k, early start time ES_k, early finish time EF_k, and disbursement rate R_k, which includes direct costs of material, labor, equipment, subcontractors, indirect costs of overheads, and taxes; $k=1,2,\ldots,n$ where n is the number of project activities (not shown in Fig. 1). Activity q, designated by a hatched bar in Fig. 1, is a representative activity of the set of activities Q_k (not shown in Fig. 1) that depend on the early finish time of activity k. Activity q has duration D_q, free float FF_q, early start time ES_q, and early finish time EF_q. Fig. 1 shows that FF_k is determined by ES_q, which indicates that ES_q is the earliest among all activities of Q_k. Activity n, designated by a solid bar in Fig. 1, is a representative activity of critical project activities at the same time as it is the terminating activity in the CPM schedule. Activity n has duration D_n, early start time ES_n, and early finish time EF_n. Duration T of the CPM schedule is the total number of working days, and i

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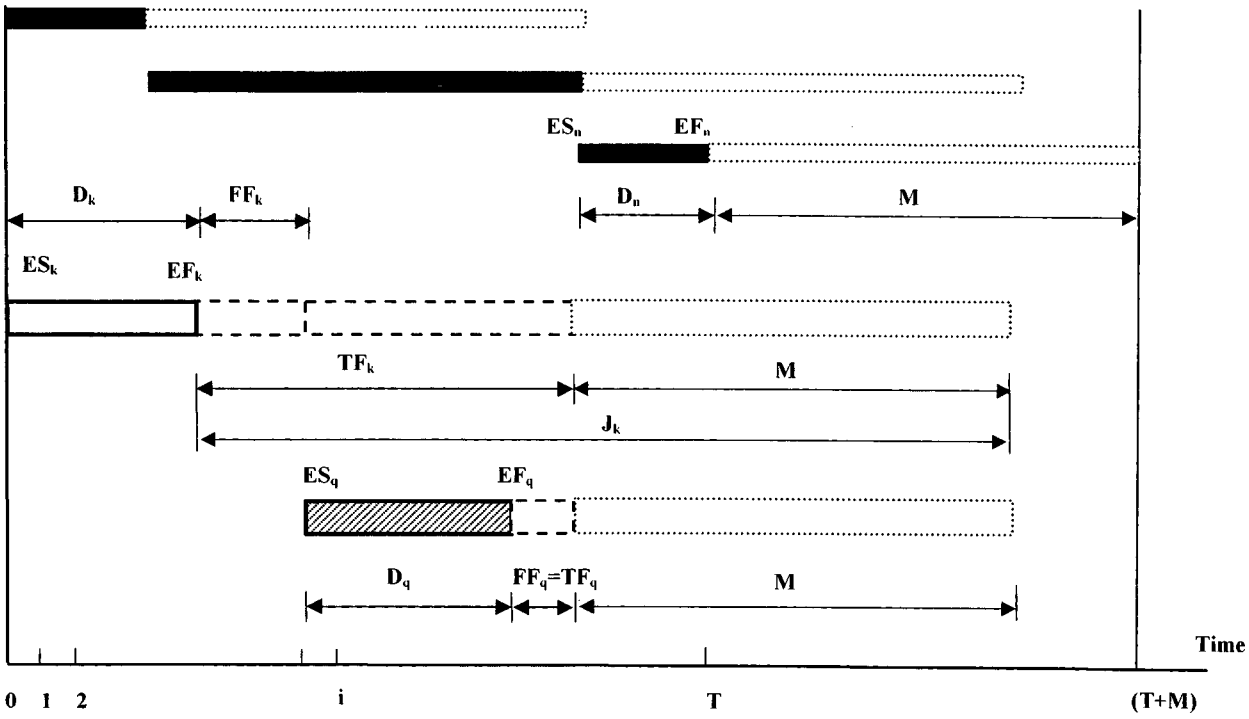

Fig. 2. Extension scheme with increment M

denotes a typical working day of the schedule. Activity q in Fig. 1 is a predecessor for critical activity n; therefore FF_q is the same as TF_q.

Devising Extension Scheme

Developing CPM schedules that are constrained by a specified credit limit involves extending rather than compressing schedules. In practice, numerous extended schedules could be produced for a given schedule; thus another fundamental objective of the method is to minimize extension in schedules. Fulfilling the objectives of devising financially feasible schedules and minimizing project duration necessitates developing an extension scheme. The extension scheme, as illustrated in Fig. 2, is a modification of the initial scheme that allows a definite extension increment M in the critical path, adds the extension increment to the original project duration to determine an extended duration $(T+M)$, and extends the total floats of activities by the extension increment to produce adjusted total floats J_k. The adjusted total float can be expressed as

$$J_k = TF_k + M \tag{1}$$

The adjusted total float is the time space within which an activity can be shifted without affecting the extended duration. For instance, in Fig. 2, activity k can be shifted all the way to the end of its illustrated adjusted total float. Consequently, activity n should be shifted to the end of its adjusted total float to maintain the relation in which activity n depends on activity k. Thus the shift of activity k can be done without causing further extension beyond the extended duration. The same scenario is valid between activities q and n.

The extension scheme allows formulating a model that simultaneously schedules activities so that negative cash flow undervalues the specified credit limit and minimizes the extension in the original CPM schedule. Thus the extension scheme transforms the process of seeking an extended schedule that fulfills financial constraints from searching in a boundless solution space to searching in a well-defined and definite solution space. Once the extension scheme is devised, the integer programming model can be formulated.

The financial terms that will be used in the model are defined in the following section. Let disbursements of all activities performed on day i be denoted by y_i; this is referred to as the project disbursement rate of day i:

$$y_i = \sum_{p=1}^{n_i} y_{pi}; \quad i = 1,2,\ldots,(T+M) \tag{2}$$

where n_i is the number of activities whose durations or adjusted total float values overlap with day i; and y_{pi} is the disbursement rate of activities in terms of the R_k values formulated later in Eqs. 16(a)–16(c).

A cash flow profile is shown in Fig. 3 for a project of L periods. The cash flow profile is plotted from the contractors' perspective. The equations in this section are presented conforming to the financial terminology used by Au and Hendrickson (1986). A project disbursement during a typical project period t is represented by E_t; the timing of receiving a receipt from the project owner is at the end of the same period; and the receipt amount is represented by P_t where

$$E_t = \sum_{i=1}^{m} y_i \tag{3}$$

$$P_t = KE_t \tag{4}$$

where m=number of days comprising a period; as indicated in Fig. 4, K=multiplier to determine the amount of payment for a given amount of disbursement. The cumulative cash flow at the end of period t (for $t \geqslant 1$) is F_t where

$$F_t = N_{t-1} + E_t \tag{5}$$

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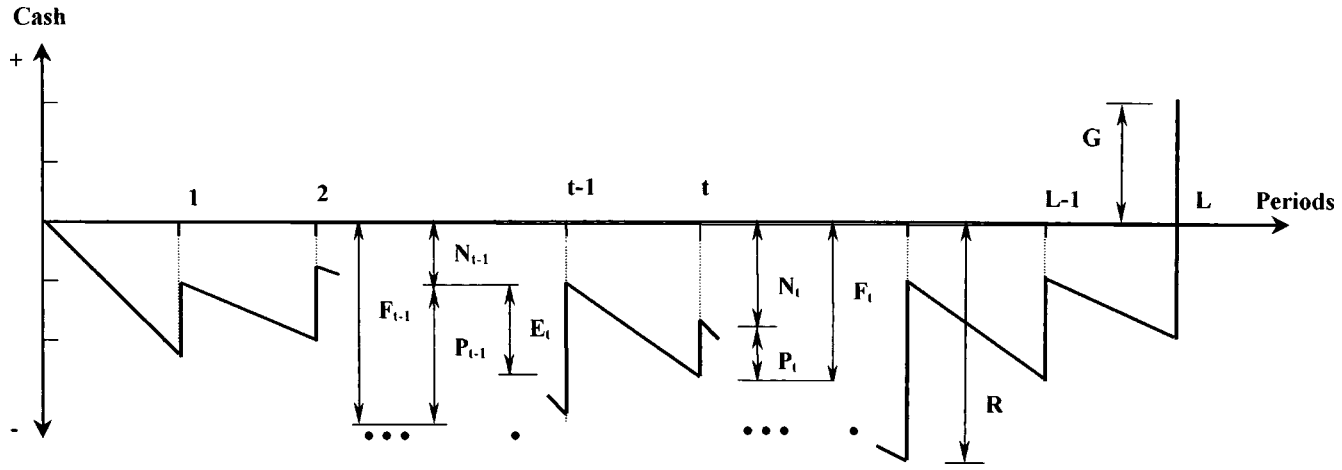

Fig. 3. Typical cash flow profile for construction project

The net cash flow at the end of period t after receiving the receipt is N_t. At the end of the previous period $(t-1)$, Fig. 3 shows the cumulative cash flow F_{t-1}, the receipt at the end of period P_{t-1}, and the net cash flows N_{t-1} where

$$N_{t-1}=F_{t-1}+P_{t-1} \tag{6}$$

In addition, Fig. 3 shows the net profit G achieved at the end of the project and the maximum cash flow R reached in this cash flow profile (not all negative cash flow values, F, are shown in Fig. 3).

Note that the above calculations are based on the assumption that the contractor pays the interest charges due at the end of each period. The total interest charges at the end of period t are I_t where

$$I_t=rN_{t-1}+r\frac{E_t}{2} \tag{7}$$

where r=interest rate per period.

The first component of I_t represents the interest per period on the net cash flow N_{t-1}; and the second component approximates the interest on E_t per period. However, if the contractor pays the periodic interest on the accumulated cash flow at interest rate r, Fig. 4 shows that the cumulative cash flow at the end of period t, which encompasses m days, including accumulated interest charges, is $\hat{F}_t$ where

$$\hat{F}_t=F_t+\hat{I}_t \tag{8}$$

The second term I_t represents the accumulated interest charges until the end of period t where

$$\hat{I}_t=\sum_{l=1}^{t} I_l(1+r)^{t-l} \tag{9}$$

Fig. 4 shows the net cash flow, including accumulated interest charges $\hat{N}_t$, until the end of period t where

$$\hat{N}_t=\hat{F}_t+P_t \tag{10}$$

Fig. 4 also shows the cumulative cash flow at the end of period $t-1$, including accumulated interest charges $\hat{F}_{t-1}$, and net cash flow $\hat{N}_{t-1}$.

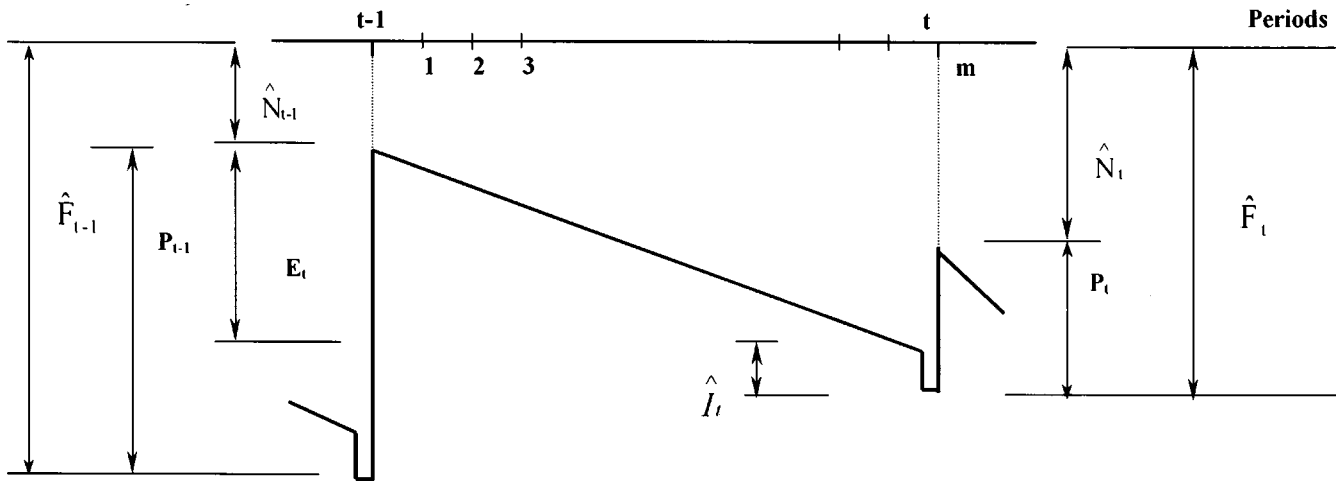

Fig. 4. Financial terms of period t considering accumulated financing cost

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Model Formulation

The model formulation stage supports building an integer programming model for the extension scheme and includes the components of formulating an objective function, as well as setting constraints such as activity shifting, activity sequence, and specified credit limit. The model solution determines the activities' shifts that minimize the total duration, fulfilling the aforementioned constraints. The model is formulated by considering the previously introduced variables and general activity k. The proposed model employed CPM constraints of activity shifting and activity sequence as developed by Easa (1989). Moreover, the present model includes an additional constraint to cover the financial aspect.

Objective Function

The decision variable x denotes the shift of an activity. As mentioned above, the objective of the model is to minimize the total extension of the schedule through minimizing the shifting of the last critical activity n. Thus the objective function can be formulated as

$$\text{minimize } z = x_n \tag{11}$$

Activity Shifting Constraints

This ensures that shifts in activities x_k; $k=1,2,\ldots,n$, where n is the total number of activities, are integers within the respective adjusted total floats. Shifts of activities x_k can be represented by

$$x_k = \sum_{j=1}^{J_k} jS_{kj}; \quad k=1,2,\ldots,n \tag{12}$$

$$\sum_{j=1}^{J_k} S_{kj} \leq 1; \quad k=1,2,\ldots,n \tag{13}$$

where J_k = adjusted float of activity k; $S_{kj} \in \{0,1\}$ are binary variables. These equations ensure that x_k takes values of $0,1,2,\ldots,J_k$, and only one of these values is considered at a time. In other words, Eq. (13) means that only one S_{kj} equals 1 and the others equal zero (in this case $x_k = j_k$), or all S_{kj} equal zero (in this case $x_k = 0$). Thus, Eqs. (12)–(13) allow x_k to take all possible values from 0 up to J_k.

Activity Sequence Constraints

A constraint between activity k and each of the activities $q \in Q_k$ is needed because the earliest finish of each of the activities q must be equal to or more than the earliest finish of activity k plus the duration of activity q, which can be represented mathematically by

$$EF_q \geq (EF_k + D_q); \quad k=1,2,\ldots,n \text{ for all } q \in Q_k \tag{14}$$

Specified Credit Limit Constraints

The negative cash flow at any period t including accumulated interest charges $\hat{F}_t$ should not exceed the specified credit limit W of the overdraft. This constraint can be formulated as follows:

$$\hat{F}_t \leq W \tag{15}$$

The term $\hat{F}_t$, as in Eq. (8), can be formulated for each period of the project in terms of disbursement rate y_{pi} using Eqs. (2)–(8), respectively. Furthermore, the disbursement rate y_{pi} resembles the formulation-of the resource rate used by Easa (1989),

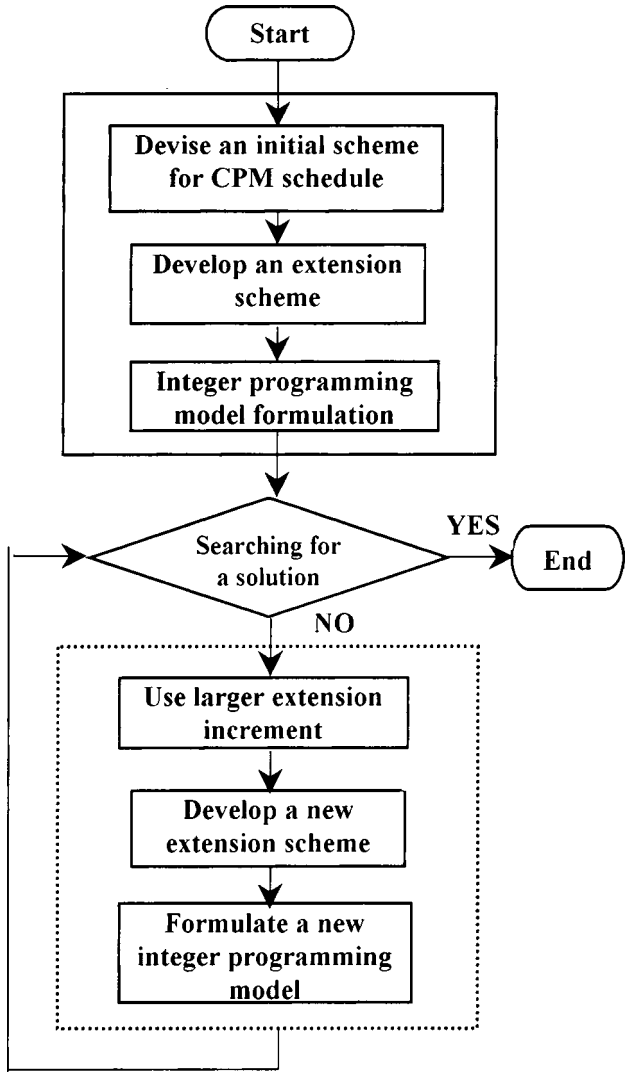

Fig. 5. Logic involved in process of searching for solution

as in Eqs. (16*a*)–(16*c*). Note that Eqs. (16*a*)–(16*c*) use the subscript k to illustrate activity k, which should be used for all activities p performed in time unit i.

For activity k of disbursement rate R_k, the disbursement rate in time unit i can be calculated as follows:

$$y_{ki} = \left(1 - \sum_{j=1}^{J_k} S_{kj}\right) R_k; \quad ES_k \leq i < EF_k \tag{16a}$$

$$y_{ki} = S_{kj}.R_k; \quad ES_k + j \leq i < EF_k + j; \quad j=1,2,\ldots,J_k \tag{16b}$$

$$y_{ki} = 0 \quad \text{otherwise} \tag{16c}$$

As noted from Eq. (16*a*), when $\Sigma S_{kj} = 0$ [$x_k = 0$ from Eq. (12)], the disbursement rate R_k will be considered for each time unit within ES_k and EF_k. As also noted from Eq. (16*b*), when any $S_{kj} = 1$ [$x_k = j_k$ from Eqs. (12) and (13)], the disbursement rate R_k will be considered for each time unit in which activity k is performed in its shifted position.

Finally, the objective function and constraints are formulated in terms of S_{kj} variables. Thus the model constraints and objective function represent an integer optimization model that can be solved directly by any of the existing computer software systems.

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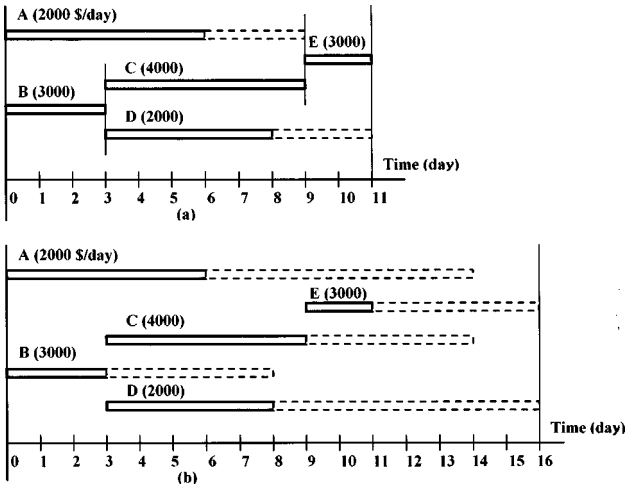

Fig. 6. Example project: (a) time-scaled diagram indicating activity direct costs; (b) extension scheme

The nonzero variable indicates shifts, for instance, $S_{k3}=1$ indicates a shift in activity k of 3 days.

Searching for Solution

The logic of searching for a solution is outlined in Fig. 5, which comprises the three stages of devising an initial scheme, devising an extension scheme, and formulating the model. A search for a solution of the model is performed, and if no solution is found, a repetition of the last two stages of the method is performed. The repetition process involves allowing a longer extension increment and consequently adjusting total floats. Another model is formulated for the new extension scheme, and a solution is sought. Thus, searching for a solution at a specified credit limit is an iterative process; each iteration uses a larger extension increment than the previous iteration. However, failure to find a solution after many iterations are performed indicates an insufficient credit limit.

Application of Method

A small example project is used to demonstrate the stages of the finance-based scheduling method presented in this paper. Fig. 6(a) shows the original schedule of a project consisting of five activities with a working day as the time unit. The direct cost per day is indicated above each activity. An extension scheme of a 5-day extension increment is shown in Fig. 6(b), as well as the corresponding adjusted total float for each activity. The extended duration is 16 working days, which represents three full periods, each comprising 5 working days and a fourth period of 1 day.

The indirect costs of the project were considered to be 10% of the direct costs and the markup to be 20% of the total cost of work accomplished during a period. As explained above, the period was selected as a calendar week of 5 working days. The payment is made 1 week after submission of the weekly pay request, with no advance payment. An interest rate of 0.3% per week was considered.

The early and late start schedules, as well as the cumulative cash flow ($\hat{F}_t$) for the project's duration, are presented in Tables 1 and 2, which indicates that the maximum cash required is \$63,998 and \$61,762, for early and late start schedules, respectively. The integer programming (IP) model was formulated for the extension scheme shown in Fig. 6(b) to devise financially feasible schedules for three credit limits of \$50,000, \$45,000, and \$40,000. The formulation of the objective function and constraints of the model with a \$50,000 credit limit is presented in the appendix.

Table 1. Example Project: Early Start and Late Start Schedules

		Early start schedule		Late start schedule	
Activity	Duration (day)	Start	Finish	Start	Finish
A	6	0	6	3	9
B	3	0	3	0	3
C	6	3	9	3	9
D	5	3	8	6	11
E	2	9	11	9	11

Note: Project duration (working days): 11

Final Project Schedule

The integer programming formulation of the above model was solved using the quantitative systems for business (QSB) computer program (Cheng 1993). Table 3 presents the results of the shift values of activities and the devised schedules at the three specified credit limits. In addition, Table 4 presents the cash flow along the project duration, and the maximum cash requirements. Note that the maximum cash requirements did not exceed the specified credit limit, and thus the 5-day extension increment was adequate to find solutions at the three specified credit limits.

Scheduling under the \$40,000 credit limit did not yield a solution, and a longer extension increment was needed to search for one. This happens because scheduling under small credit limits creates bottlenecks that take place when the available cash at some period does not allow for any work to proceed. In practice, these bottlenecks can be resolved either by increasing the project credit limit or by seeking short-term loans for these periods. Although scheduling of this example project was based on an overdraft with a credit limit at a fixed level for all periods, it still can be done with credit limits at different levels.

Computer Implementation

The integer programming model provides an efficient means of devising finance-based schedules. This model requires as input the original CPM scheduling results, the payment terms of the project contract, and cost-estimating information. However, the process of formulating constraints and objective functions is time

Table 2. Example Project: Cumulative Cash Flow along the Project Duration

	Cumulative cash flow before receipt of weekly payment (\$)	
End of week	Early start schedule	Late start schedule
1	(−34,151)	(−23,135)
2	(−63,998)	(−61,762)
3	(−30,557)	(−42,433)
4	1,524	(−855)
Maximum cash required (\$)	(−63,998)	(−61,762)

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Table 3. Example Project: Summary of Scheduling Results at Three Credit Limits—shifting Start of Activities and Project Duration (Working Days)

		Credit limit of $50,000			Credit limit of $45,000			Credit limit of $40,000		
Activity	Duration (day)	Shift	Start	Finish	Shift	Start	Finish	Shift	Start	Finish
A	6	0	0	6	0	0	6	0	0	6
B	3	1	1	4	2	2	5	2	2	5
C	6	2	5	11	3	6	12	5	8	14
D	5	5	8	13	6	9	14	8	11	16
E	2	2	11	13	3	12	14	5	14	16
Project duration (working days)			13			14			16	

Table 4. Example Project: Summary of Scheduling Results at Three Credit Limits—Cash Flow along the Project Duration

	Cumulative cash flow before receipt of weekly payment ($)		
End of week	Credit limit of $50,000	Credit limit of $45,000	Credit limit of $40,000
1	(−20,931)	(−20,931)	(−20,931)
2	(−49,637)	(−43,027)	(−32,011)
3	(−44,773)	(−44,753)	(−39,211)
4	(−13,926)	(−21,056)	(−32,922)
5	—	—	(−848)
Maximum cash required ($)	(−49,637)	(−44,753)	(−39,211)

consuming and prone to error. Thus, a prototype interface program was developed by the writers using macros and templates by Microsoft Excel 97, a popular commercial spreadsheet program. The development of the interface program encourages the use of integer programming by providing a user-friendly template for data entry and macros for formulating constraints and objective function. Users can enter precedence relationships, durations, and cost information for activities similar to defining CPM schedules. The interface program then automatically formulates the objective function and constraints for the model as an output, which is then input to an optimization program that solves the model. The interface and optimization programs can be chained to existing CPM scheduling packages so that financially feasible schedules can be produced with little extra effort. The template for data entry and output of the interface program is illustrated in the following section using the data from the example project.

The data required to run the interface program are shown in Fig. 7, which represents a template of three portions of input data. Column A, ''Activity,'' of the first portion is used to identify the activities of the project; column B, ''Duration,'' is the activity

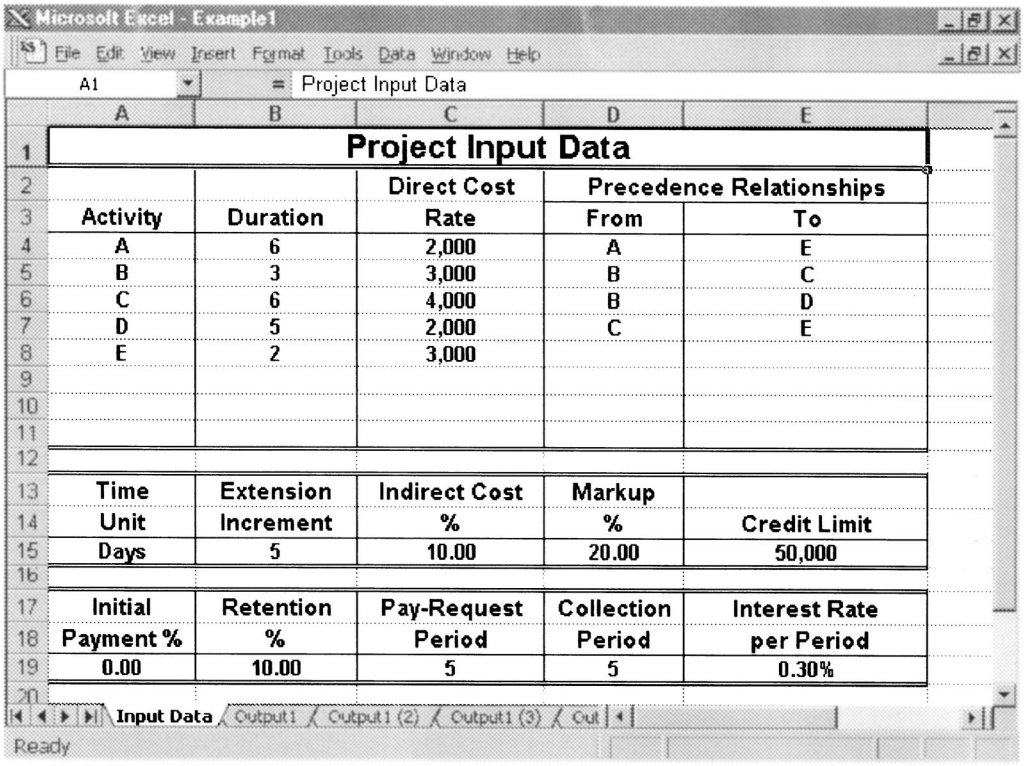

Fig. 7. Input data template of interface program

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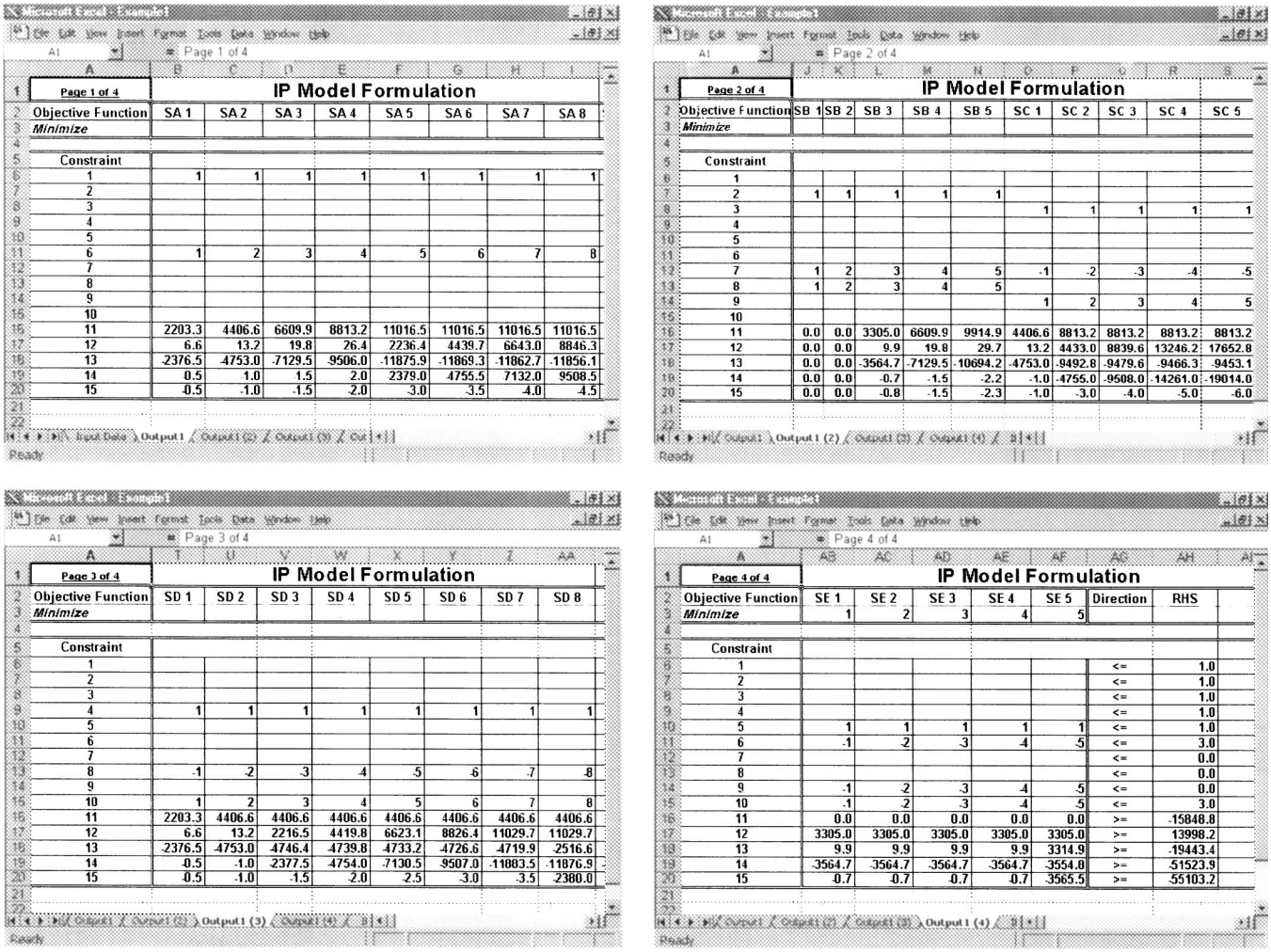

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IP Model Formulation

Objective Function	SA 1	SA 2	SA 3	SA 4	SA 5	SA 6	SA 7	SA 8
Minimize								
Constraint								
1	1	1	1	1	1	1	1	1
2								
3								
4								
5								
6	1	2	3	4	5	6	7	8
7								
8								
9								
10								
11	2203.3	4406.6	6609.9	8813.2	11016.5	11016.5	11016.5	11016.5
12	6.6	13.2	19.8	26.4	2236.4	4439.7	6643.0	8846.3
13	-2376.5	-4753.0	-7129.5	-9506.0	-11875.9	-11869.3	-11862.7	-11856.1
14	0.5	1.0	1.5	2.0	2379.0	4755.5	7132.0	9508.5
15	-0.5	-1.0	-1.5	-2.0	-3.0	-3.5	-4.0	-4.5

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IP Model Formulation

Objective Function	SB 1	SB 2	SB 3	SB 4	SB 5	SC 1	SC 2	SC 3	SC 4	SC 5
Minimize										
Constraint										
1										
2	1	1	1	1	1					
3						1	1	1	1	1
4										
5										
6										
7	1	2	3	4	5	-1	-2	-3	-4	-5
8	1	2	3	4	5					
9						1	2	3	4	5
10										
11	0.0	0.0	3305.0	6609.9	9914.9	4406.6	8813.2	8813.2	8813.2	8813.2
12	0.0	0.0	9.9	19.8	29.7	13.2	4433.0	8839.6	13246.2	17652.8
13	0.0	0.0	-3564.7	-7129.5	-10694.2	-4753.0	-9492.8	-9479.6	-9466.3	-9453.1
14	0.0	0.0	-0.7	-1.5	-2.2	-1.0	-4755.0	-9508.0	-14261.0	-19014.0
15	0.0	0.0	-0.8	-1.5	-2.3	-1.0	-3.0	-4.0	-5.0	-6.0

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IP Model Formulation

Objective Function	SD 1	SD 2	SD 3	SD 4	SD 5	SD 6	SD 7	SD 8
Minimize								
Constraint								
1								
2								
3								
4	1	1	1	1	1	1	1	1
5								
6								
7								
8	-1	-2	-3	-4	-5	-6	-7	-8
9								
10	1	2	3	4	5	6	7	8
11	2203.3	4406.6	4406.6	4406.6	4406.6	4406.6	4406.6	4406.6
12	6.6	13.2	2216.5	4419.8	6623.1	8826.4	11029.7	11029.7
13	-2376.5	-4753.0	4746.4	-4739.8	4733.2	-4726.6	-4719.9	-2516.6
14	-0.5	-1.0	-2377.5	-4754.0	-7130.5	-9507.0	-11883.5	-11876.9
15	-0.5	-1.0	-1.5	-2.0	-2.5	-3.0	-3.5	-2380.0

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IP Model Formulation

Objective Function	SE 1	SE 2	SE 3	SE 4	SE 5	Direction	RHS
Minimize	1	2	3	4	5		
Constraint							
1						<=	1.0
2						<=	1.0
3						<=	1.0
4						<=	1.0
5	1	1	1	1	1	<=	1.0
6	-1	-2	-3	-4	-5	<=	3.0
7						<=	0.0
8						<=	0.0
9	-1	-2	-3	-4	-5	<=	0.0
10	-1	-2	-3	-4	-5	<=	3.0
11	0.0	0.0	0.0	0.0	0.0	>=	-15848.8
12	3305.0	3305.0	3305.0	3305.0	3305.0	>=	13998.2
13	9.9	9.9	9.9	9.9	3314.9	>=	-19443.4
14	-3564.7	-3564.7	-3564.7	-3564.7	-3554.8	>=	-51523.9
15	-0.7	-0.7	-0.7	-0.7	-3565.5	>=	-55103.2

Fig. 8. IP model formulation for example project

duration in working days; column C, ''Direct Cost Rate,'' is the direct cost per day of each activity; and columns D and E are used to define the precedence relationships between activities. In this example, activity E cannot start until activities A and C are completed. Column A, ''Time Unit,'' of the second portion of the figure, is used to identify the unit of the activity duration; column B represents the extension increment; column C is the indirect cost rate, which indicates the indirect cost percentage of the direct costs; column D is the markup as a percentage of total costs; and column E specifies the credit limit. In the third portion of the figure, columns A, B, C, and D indicate the value of the initial payment as a percentage of the total project price, the retention percentage of each pay request, the period of submitting a pay request, and the collection period of a pay request, respectively, and column (E) indicates the interest rate per period.

Once data are input, the interface program is run to formulate the objective function and constraints. The output is shown in Fig. 8, which is composed of four Excel screens arranged horizontally. Fig. 8 puts the 31 decision variables of the model along the row tilted ''Objective Function'' and tabulates the coefficients of each variable in the objective function along the row tilted ''Minimize.'' Variable coefficients, inequality signs, and right-hand sides (RHS) of constraints are given in rows 1 to 15.

The formulated objective function and constraints are input to the QSB. The exact solution can be generated by using integer programming. The elapsed CPU time to find the exact solution is 46.29 s for the example network on a Pentium II-266 MHz PC. Fig. 9 shows the output of the QSB integer programming to solve the example project.

Future Development

Future developments of the interface program include (1) linking existing project schedule applications, such as Microsoft Project

Solution Summary for Example1

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Variable	Solution	Objective Coefficient	Variable	Solution	Objective Coefficient
B1	1	0	E2	1	2
C2	1	0	Limit	50000	0
D5	1	0			

Minimized OBJ = 2 Iteration = 502 Elapsed CPU seconds = 46.2968
Branch selection: Newest problem Integer tolerance = .01 Max. #node = 5

< PageDown > < PageUp > < Hardcopy > < Cancel >

Fig. 9. Quantitative systems for business output of example project

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and Primavera, with the program data input template so that schedule activity identification, precedence relationships, duration, and cost can be directly imported into the interface program; (2) exporting the objective function and constraints directly to optimization software, thus preventing double handling of data; and (3) modifying the results of the scheduling software according to the output of the optimization software.

Conclusion

This paper presents a method for devising financially feasible schedules that balances work with the available cash to perform it. The method offers a realistic and useful concept of scheduling construction projects; builds on the widely spread CPM technique and thus is readily plausible for practitioners; builds trust in CPM schedules and thus increases the willingness to use the CPM as a project management tool; produces realistic schedules as far as finance availability is concerned and thus increases the utility of schedules; achieves financial feasibility besides the demanded goal of time minimization; employs the optimization technique of integer programming to give optimum solutions; defines an extension scheme that transforms the process of seeking an extended schedule that fulfills finance constraints from searching in a boundless solution space to searching in a well-defined and definite solution space; provides a sensitivity analysis technique through searching incrementally increasing solution spaces; provides an invaluable tool for negotiating and establishing good bank overdrafts; and brings business and production personnel to a common ground of understanding so that their goals are unified rather than conflicting. In addition, this paper presents a spreadsheet interface program to formulate the model objective function and constraints. The IP model, along with the spreadsheet tool, further improves the performance of CPM by considering the major concern of financing.

Appendix. Model Formulation of Example Project

Objective Function

Minimize $Z=S_{E1}+2S_{E2}+3S_{E3}+4S_{E4}+5S_{E5}$ subject to

- Activity shifting constraints

$$S_{A1}+S_{A2}+S_{A3}+S_{A4}+S_{A5}+S_{A6}+S_{A7}+S_{A8}\leqslant 1 \quad (17)$$

$$S_{B1}+S_{B2}+S_{B3}+S_{B4}+S_{B5}\leqslant 1 \quad (18)$$

$$S_{C1}+S_{C2}+S_{C3}+S_{C4}+S_{C5}\leqslant 1 \quad (19)$$

$$S_{D1}+S_{D2}+S_{D3}+S_{D4}+S_{D5}+S_{D6}+S_{D7}+S_{D8}\leqslant 1 \quad (20)$$

$$S_{E1}+S_{E2}+S_{E3}+S_{E4}+S_{E5}\leqslant 1 \quad (21)$$

- Activity sequence constraints

$$S_{A1}+2S_{A2}+3S_{A3}+4S_{A4}+5S_{A5}+6S_{A6}+7S_{A7}+8S_{A8}-S_{E1}-2S_{E2}-3S_{E3}-4S_{E4}-5S_{E5}\leqslant 3 \quad (22)$$

$$S_{B1}+2S_{B2}+3S_{B3}+4S_{B4}+5S_{B5}-S_{C1}-2S_{C2}-3S_{C3}-4S_{C4}-5S_{C5}\leqslant 0 \quad (23)$$

$$S_{B1}+2S_{B2}+3S_{B3}+4S_{B4}+5S_{B5}-S_{D1}-2S_{D2}-3S_{D3}-4S_{D4}-5S_{D5}-6S_{D6}-7S_{D7}-8S_{D8}\leqslant 0 \quad (24)$$

$$S_{C1}+2S_{C2}+3S_{C3}+4S_{C4}+5S_{C5}-S_{E1}-2S_{E2}-3S_{E3}-4S_{E4}-5S_{E5}\leqslant 0 \quad (25)$$

$$S_{D1}+2S_{D2}+3S_{D3}+4S_{D4}+5S_{D5}+6S_{D6}+7S_{D7}+8S_{D8}-S_{E1}-2S_{E2}-3S_{E3}-4S_{E4}-5S_{E5}\leqslant 3 \quad (26)$$

- Specified credit limit constraints

$\hat{F}_1$ (first period)

$$2203.3S_{A1}+4{,}406.6S_{A2}+6{,}609.9S_{A3}+8{,}813.2S_{A4}+11{,}016.5S_{A5}+11{,}016.5S_{A6}+11{,}016.5S_{A7}+11{,}016.5A_{A8}+3{,}305.0S_{B3}+6{,}609.9S_{B4}+9{,}914.9S_{B5}+4{,}406.6S_{C1}+8{,}813.2S_{C2}+8{,}813.2S_{C3}+8{,}813.2S_{C4}+8{,}813.2S_{C5}+2{,}203.3S_{D1}+4{,}406.6S_{D2}+4{,}406.6S_{D3}+4{,}406.6S_{D4}+4{,}406.6S_{D5}+4{,}406.6S_{D6}+4{,}406.6S_{D7}+4{,}406.6S_{D8}\geqslant -15{,}848.8 \quad (27)$$

$\hat{F}_2$ (second period)

$$6.6S_{A1}+13.2S_{A2}+19.8S_{A3}+26.4S_{A4}+2{,}236.4S_{A5}+4{,}439.7S_{A6}+6{,}643.0S_{A7}+8{,}846.2S_{A8}+9.9S_{B3}+19.8S_{B4}+29.7S_{B5}+13.2S_{C1}+4{,}433.0S_{C2}+8{,}839.6S_{C3}+13{,}246.2S_{C4}+17{,}652.8S_{C5}+6.6S_{D1}+13.2S_{D2}+2{,}216.5S_{D3}+4{,}419.8S_{D4}+6{,}623.1S_{D5}+8{,}826.4S_{D6}+11{,}029.7S_{D7}+11{,}029.7S_{D8}+3{,}305.0S_{E1}+3{,}305.0S_{E2}+3{,}305.0S_{E3}+3{,}305.0S_{E4}+3{,}305.0S_{E5}\geqslant 13{,}998.2 \quad (28)$$

$\hat{F}_3$ (third period)

$$-2{,}376.5S_{A1}-4{,}753.0S_{A2}-7{,}129.5S_{A3}-9{,}506.0S_{A4}-11{,}875.9S_{A5}-11{,}869.3S_{A6}-11{,}862.7S_{A7}-11{,}856.1S_{A8}-3{,}564.7S_{B3}-7{,}129.5S_{B4}-10{,}694.2S_{B5}-4{,}753.0S_{C1}-9{,}492.8S_{C2}-9{,}479.6S_{C3}-9{,}466.3S_{C4}-9{,}453.1S_{C5}-2{,}376.5S_{D1}-4{,}753.0S_{D2}-4{,}746.4S_{D3}-4{,}739.8S_{D4}-4{,}733.2S_{D5}-4{,}726.6S_{D6}-4{,}719.9S_{D7}-2{,}516.6S_{D8}+9.9S_{E1}+9.9S_{E2}+9.9S_{E3}+9.9S_{E4}+3{,}314.9S_{E5}\geqslant -19{,}443.4 \quad (29)$$

$\hat{F}_4$ (fourth period)

$$-0.5S_{A1}-1.0S_{A2}-1.5S_{A3}-2.0S_{A4}-2{,}379.0S_{A5}-4{,}755.5S_{A6}-7{,}132.0S_{A7}-9{,}508.5S_{A8}-0.7S_{B3}-1.5S_{B4}-2.2S_{B5}-1.0S_{C1}-4{,}755.0S_{C2}-9{,}508.0S_{C3}-14{,}261.0S_{C4}-19{,}014.0S_{C5}-0.5S_{D1}-1.0S_{D2}-2{,}377.5S_{D3}-4{,}754.0S_{D4}-7{,}130.5S_{D5}-9{,}507.0S_{D6}-11{,}883.5S_{D7}-11{,}876.9S_{D8}-3{,}564.7S_{E1}-3{,}564.7S_{E2}-3{,}564.7S_{E3}-3{,}564.7S_{E4}-3{,}554.8S_{E5}\geqslant -51{,}523.9 \quad (30)$$

$\hat{F}_5$ (fifth period)

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$$-0.5S_{A1}-1.0S_{A2}-1.5S_{A3}-2.0S_{A4}-3.0S_{A5}-3.5S_{A6}-4.0S_{A7}$$
$$-4.5S_{A8}-0.8S_{B3}-1.5S_{B4}-2.3S_{B5}-1.0S_{C1}-3.0S_{C2}$$
$$-4.0S_{C3}-5.0S_{C4}-6.0S_{C5}-0.5S_{D1}-1.0S_{D2}-1.5S_{D3}$$
$$-2.0S_{D4}-2.5S_{D5}-3.0S_{D6}-3.5S_{D7}-2{,}380.0S_{D8}-0.7S_{E1}$$
$$-0.7S_{E2}-0.7S_{E3}-0.7S_{E4}-3{,}565.5S_{E5}\geqslant -55{,}103.2 \quad (31)$$

Notation

The following symbols are used in this paper:

D_k = duration of activity k;
E_t = construction disbursement in period t;
EF_k = early finish time of activity k;
ES_k = early start time of activity k;
$\hat{F}_t$ = cumulative cash flow at end of period t, including accumulated interest charges;
F_t = cumulative cash flow at end of period t just before receiving payment P_t;
FF_k = free float of activity k;
G = net profit achieved at end of project;
$\hat{I}_t$ = accumulated interest charges till end of period t;
I_t = total interest charges at end of period t;
J_k = adjusted float of activity k;
M = extension increment for critical path;
m = number of days comprising period;
$\hat{N}_t$ = cumulative net cash flow until period t, including accumulated interest charges;
N_t = cumulative net cash flow until period t;
n = total number of activities;
n_i = number of activities having durations and J_k values overlapping time unit i;
Q_k = set of activities immediately following activity k;
P_t = receipt from owner payment at end of period t;
R = maximum cash flow reached in cash-flow profile;
R_k = disbursement rate of activity k (\$/day);
r = interest rate per period;
S_{kj} = binary variables, that is, $S_{kj} \in \{0,1\}$;
T = total project duration;
TF_k = total float of activity k;
W = available fund or credit limit;
x_k = amount of shifting of activity k;
y_i = disbursement rate of all activities performed in time unit i;
y_{ki} = modeling disbursement rate of activity k in time unit i in terms of R_k; and
z = value of objective function of model.

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