

Review of Unbalanced Bidding Models in Construction

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Abstract: Unbalanced bidding describes an activity otherwise known as item price loading. It is a practice used to some extent by building contractors to determine the prices that they will allocate to the individual component items within a project. This practice takes advantage of the contractor's opportunity to manipulate these prices without this affecting their overall bid price for a project. Three types of loading are described, namely those of "front-end loading," "back-end loading," and "quantity error exploitation" (otherwise known as "individual rate loading"). Several scientists have expressed an interest in this field, starting with Marvin Gates in 1959. All of these scientific endeavors have entailed the attempt by which to mathematically determine the optimum method of item pricing. These efforts are seen as potentially significant given that it has become recognized that this practice can contribute substantially to a contractor's profit as well as their risk. It is therefore interesting to note that although some research has found that some forms of unbalanced bidding are being practiced, there is no published research known to the writers that describes any practical application of any of the mathematical models that have been advocated by the aforementioned scientific community. A critical assessment is made of all of the scientific contributions known to the writers that have been in this field. This critical assessment finds flaws with all of the existing scientific proposals. In particular it finds fault with the technique that is incorporated into many of them by which prices are bounded by arbitrary upper and lower limits that have no scientific basis. It also concludes that further research is required to test the practical efficacy of some of these academic models.

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Introduction

Unbalanced bidding models are mathematical techniques for use by vendors for the purposes of benefiting from the uneven distribution of markup among a project's component items. They are typically designed for use by building or engineering contractors and are also often intended for use in the oil and forestry industries. This paper is, however, concerned primarily with the use of these models by contractors in the construction industry although much of this research is relevant to the full spectrum of the potential use of these models.

Typically a contractor has to decide the individual prices of all of a project's component items so that in summation they are equal to the tender price. This presents the contractor with the opportunity [of which some avail themselves, according to

McCaffer (1979); Green (1986); Kaka and Price (1991); Kenley (2003)] to price some items using a high markup, and others using a low markup (to compensate). The incentive for this uneven distribution of markup relies on the role by which the individual item prices form part of the contract and by which they are used for the valuation of interim payments, escalation, compensation, and quantity variations. Contractors are potentially able to enjoy greater profits from this manipulation of their prices although it is also possible that it may give cause for them to incur additional risk. It is, however, possible that contractors can use it as a technique by which to reduce their risk.

An example of unbalanced bidding would be if a contractor were to apply high prices to all of the items that are scheduled to arise early in a project. The effect of this would be for the client to pay the contractor far more in the early stages of a contract than if the bid were instead a balanced one. These boosted early payments can be of considerable value (Kenley 2003) to a contractor in terms of providing the working capital requirements of the contractors.

The objective of unbalanced bidding is for a contractor to derive some advantage that they would not enjoy from a balanced bid. This will often, but not always, occur at the expense of the client—most probably without the client initially being aware that they are being given this exposure. Contractors have the advantage over clients in this regard by having available to them their own confidential information about their estimates of a project's cost. Clients do not have access to this same information, which makes it far more difficult for them to determine the existence and/or the extent of any possible loading in any item prices.

Contractually, clients will typically, in terms of traditional building practice, provide a specification and design of the project that they wish to have built. Contractors commit themselves

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legally to fulfilling these requirements, but they are left entitled to decide their own approach and technique by which they will accomplish this. This power of discretion that is enjoyed by contractors is significant here from the perspective that different contractors are entitled to choose different methods of construction that might lead to different costs. Furthermore, different contractors have different resources available to them, again with corresponding differences in cost. For example, one contractor might have available for a project some plant, such as a crane, which may be very well suited to a project and in which their long-standing investment is already "written off" in their books giving them a cost advantage over their competitors. Different contractors have different suppliers and subcontractors or at least might have different relationships with these. Thus, different contractors will incur different costs if they were all to build the same project, even if they were to build it at the exact same time, with the exact same set of imposed conditions (such as the weather). Contractors also differ from one another by way of the techniques that they use by which they estimate these costs and thus different contractors, while competing for the same job, might reasonably be expected to have estimated item costs that are quite different from one another. Research by Beeston (1975) has shown that the variance between contractors in individual, component, estimated item costs far exceeds the overall variance that lies between contractor's overall, composite, estimated project costs.

With these variances being in existence, a client might have difficulty being able to differentiate between a balanced set of item prices and an unbalanced one. Furthermore, if one recognizes from the above that contractors are not all operating off the exact same set of estimated costs, why is it ethically significant that they should be expected to present a balanced bid? Should contractors not be entitled to price different items with different markups if only so as simply to recognize their relative, competitive differences from other contractors, with respect to any such items? For example, if their estimated cost of their crane were extremely low, perhaps for the reason that it was purchased many years ago, could it not be justified that they might apply a high markup to their unusually low cost?

A further consideration with regard to the ethics of unbalanced bidding is that a client is given a full disclosure of a contractor's item prices and hence is given the choice, prior to having to finally accept any such bid and prior to having to decide on the final choice of the contractor, in preference to any of their competitors.

When contractors submit their item prices they do so without any inherent assurance that these prices constitute a balanced bid. They are asked to present their prices and not their calculations by which they have arrived at these prices. When submitting these prices they do so knowing the risk that they may be found to be unacceptable. They could be rejected as much for the reason that they were derived from a peculiar cost estimate as for the reason that they are derived from an uneven distribution of the markup.

As much as a client has no right to dictate to a contractor how they should build a project or how they should estimate their costs, it follows that they have no right to dictate to a contractor how they should price the project either. The client does, however, retain the right to reject outright any prices that a contractor submits to them. To this end, contractors need to consider this risk when pricing a project.

Three Types of Unbalanced Bidding

Currently, it is believed that there are three different, complementary approaches to unbalanced bidding [see Cattell (1984, 1987); Green (1989); Cattell et al. (2004)], namely, front-end loading, individual rate loading, and back-end loading.

"Front-end loading" refers to the loading up of the prices of items that will arise early in the construction schedule, which will obviously improve a contractor's cashflow.

"Back-end loading," on the other hand, is designed to take advantage of the contractual mechanism by which contractors may be compensated for inflationary increases in their expenses by way of escalation. Back-end loading is the process of loading the prices of items that will arise late in the schedule. It can also entail loading those items that fall within workgroups (that is, categories of work) that are expected to have a comparatively high rate of escalation.

The third style of loading is called "quantity error exploitation" or otherwise as "individual rate loading." This process entails loading the price of items whose final quantities are expected to exceed the initial quantities contained in the tender documentation.

Gates' Strategy

It is believed that Marvin Gates (1959, 1967) was the first to comment on the practice of unbalanced bidding. Gates' entry into this arena has largely been deemed significant because he proposed an alternative to Friedman's (1956) (balanced) bidding model, subsequently coming to criticize the latter (when he became aware of it) for what he considered to be an incorrect use of mathematics (Gates 1970). Friedman's model pioneered the concept that a contractor might apply mathematics for the purpose of determining the probability of winning a closed tender bid. This mathematical model (being a "balanced bidding model" by definition) entails analysis of a contractor's competitive environment on the basis that: (1) ideally, the contractor has knowledge of who the competitors are; or (2) it is only known how many competitors there are; or (3) worst of all, it is not even known how many competitors there are. Friedman's model provides a basis by which a contractor can utilize this analysis by which to determine an optimum bid.

Gates' efforts were largely focused on proposing an alternative mathematical formula by which to determine this probability. The Gates versus Friedman debate has raged on ever since with many dozens of researchers falling largely into one or the other of these two camps [see, for example, Skitmore (2002)]. Most bidding models are derived from either Friedman's or Gates' original models. Abdel-Razek (1987) is one of those who have provided a synopsis of the early stages of this conceptual "battle," Crowley (2000) provides a more recent assessment, and Skitmore (2002) provides a quantitative comparison.

While the principal area of interest with regard to Gates' research has been with respect to this method of determination of the probability of winning (and hence the determination of an optimum project markup), another proposal of his went less well noticed. He is the first researcher to have commented upon the concept of unbalanced bidding (Gates 1959). In the process he came to suggest that unbalanced bidding had more to offer a contractor in the short term as a strategy than any other bidding strategy (Gates 1959).

Table 1. Gates' Example of Balanced Bid [Reprinted with Permission from Gates (1967)]

Item number	Description	Unit	Quantity	Unit bid (dollars)	Amount (dollars)
1	Clearing	Lump sum	As necessary	50,000.00	50,000
2	Earth excavation	Cubic yards	50,000	1.50	75,000
3	Rock excavation	Cubic yards	25,000	3.00	75,000
4	Cleaning up	Lump sum	As necessary	50,000.00	50,000
Total bid					250,000

He proposed a method that addressed a contractor's need for an accelerated cashflow as well as a means to benefit from loading the prices of items whose quantities are anticipated to increase as a result of a likely variation order. His approach largely steered clear of any sophisticated mathematics and he went so far as to comment that he felt that unbalanced bidding, at least insofar as the manner in which he advocated it, was the least mathematically involved of all the bidding strategies that he was then proposing.

Gates (1967) used the simple four-item project shown in Table 1 to illustrate this strategy.

Gates proposed that while Table 1 illustrates the item breakdown of a \$250,000 project, as it might be without any loading, Table 2 illustrates how it might appear if its items were given the treatment of front-end loading.

Gates hence showed how this project could be made to generate an additional \$40,000 early on in the project, which clearly is of value to the contractor to finance his operations.

Gates then went on to suggest that another method of unbalanced bidding was to exploit obvious errors in the project's initial quantities. He referred to this technique as "unclassifying." He illustrated this with the example of a contractor, having done a field investigation, "having reason to believe" that instead of 50,000 "cubic yards" (yd³) of earth excavation, there is more likely to be 70,000 yd³; and instead of 25,000 yd³ of rock excavation, there is more likely to be only 5,000 yd³. He suggested that this contractor would benefit if they were to load up the price of earth excavation, and load down the price of rock excavation. He illustrated this with the following example, shown in Table 3.

The rate for excavation of \$1.60 was arrived at solely for the reason that he proposed that custom dictated that the rate for rock excavation could not be less than the rate for earth excavation. The rate was calculated as the weighted average based on the contractor's "best estimate" of the final quantities of the two items of excavation, as follows:

$$\frac{(70,000 \times \$1.50) + (5,000 \times \$3.00)}{75,000} = \$1.60/\text{yd}^3$$

Gates' example drew comparison with the (unaltered) balanced bid, as shown in Table 4. Gates then showed the obvious benefits of his proposed "classified bid," depicted in Table 5.

Thus Gates showed that while both his examples of a balanced bid and a classified bid comprised the same tender price of \$250,000, the latter unbalanced bid would generate an additional \$30,000 for the contractor provided that his expectation of the variations to the project's earthworks were to prove correct.

Gates commented that there was, however, considerable risk for a contractor that his predictions regarding the anticipated variations may not be fulfilled. Notwithstanding this caveat, he did not propose any basis by which to measure or address this risk.

Gates' work is believed to be important for reason of having identified unbalanced bidding as a significant strategy. Gates did not, however, succeed with regard to identifying any mathematical techniques by which to accomplish the potential of this strategy. He also did not recognize the potential for the use of sophisticated mathematics for this purpose (Gates 1967).

Stark's Model

Stark (1968, 1972, 1974) approached the problem largely as Gates had defined it. Unlike Gates, however, he recognized that there was potential for the application of more sophisticated mathematics and his approach advocated a linear programming solution.

Stark (1968) recognized that there are the following constraints that govern item pricing:

1. **Bid constraint.** This constraint simply ensures that all of the items' prices add up to the tender price, which may be stated as follows:

$$TP = \sum_{j=1}^J Q_j P_j \quad (1)$$

where j =item number; J =number of items; Q_j =bill quantity of item j ; P_j =bill price per unit of item j ; and TP =tender price.

Table 2. Gates' Example of Front-End Loading [Reprinted with Permission from Gates (1967)]

Item number	Description	Unit	Quantity	Unit bid (dollars)	Amount (dollars)
1	Clearing	Lump sum	As necessary	90,000.00	90,000
2	Earth excavation	Cubic yards	50,000	1.50	75,000
3	Rock excavation	Cubic yards	25,000	3.00	75,000
4	Cleaning up	Lump sum	As necessary	10,000.00	10,000
Total bid					250,000

Table 3. Gates' Example of Bid Incorporating Quantity Error Exploitation [Reprinted with Permission from Gates (1967)]

Item number	Description	Unit	Quantity	Unit bid (dollars)	Amount (dollars)
1	Clearing	Lump sum	As necessary	50,000.00	50,000
2	Earth excavation	Cubic yards	50,000	1.60	80,000
3	Rock excavation	Cubic yards	25,000	1.60	40,000
4	Cleaning up	Lump sum	As necessary	50,000.00	50,000
Total bid					250,000

2. **Unit bid constraints.** Stark suggested that, for reason of "custom," some items' prices are governed by other items' prices while some simply need to be bounded by upper and lower bounds that are seemingly arbitrarily decided upon.

These two constraints may be expressed as follows:

$$P_i - P_j \geq 0 \quad (2)$$

and

$$P_{jl} \leq P_j \leq P_{ju} \quad (3)$$

where i, j =item numbers; and P_{jl}, P_{ju} =lower and upper bounds, respectively, for the price of item j .

Stark suggested, as an example, that custom dictated that the excavation of hard rock should be priced more than the excavation of soft rock, which in turn should be priced more than the excavation of earth. He made no attempt, however, to provide any scientific justification for this nature of constraint. Presumably he meant only for this constraint to prevent the situation where otherwise it would draw attention to the client that the contractor had manipulated the prices.

He also proposed that the upper and lower bounds would be especially useful to limit a contractor's risk where the contractor felt less certain of the final quantity of an item. He did not, however, propose any scientific basis by which to decide these bounds nor did he suggest a scientific basis by which to identify the items to which to apply this technique.

3. **Rate constraints.** Stark went on to propose that a contractor may wish to ensure that their anticipated receipts of interim progress payments remain in some proportion to the rate at which the work is being done. He suggested that this constraint be described by way of the following formula:

$$\frac{m}{N} \sum_{j=1}^J Q_j P_j = \alpha \sum_{j=1}^J \sum_{n=1}^m Q_{nj} P_j \quad (4)$$

where m =month in the range between 1 and N ; N =estimated duration of the project (months); α =Stark's "constant of

proportionality"; and Q_{nj} =quantity of item j expected to be built in month n .

His suggestion was that a contractor should have an "intuitive" feel for deciding an appropriate value of α and that perhaps he should test a range of values within some bounded limits (also to be arbitrarily decided upon). He indicated that a value of 1 should have the effect that interim payments should keep track with the rate of progress, such that when, for example, 20% of the project is completed (measured in terms of value completed, as determined by way of the project's component item prices) then 20% of the overall project's value would be paid to the contractor. Values higher than 1 should have the effect of front-end loading (i.e., that of accelerating the payments) while values of less than 1 should give cause for (a basic form of) back-end loading (i.e., that of delaying the payments—not that this was something he advocated).

In effect though, the formulation of this constraint appears fundamentally flawed. If this constraint were to be replicated for all months m then the only basis by which all these constraints could be satisfied as equalities is in the event of the special case of α being equal to 1 and where the rate of progress, and hence the rate of interim progress payments, of the project was exactly linear. Besides the impracticality of the latter assumption, this then also negates the effective use of α for the purpose for which Stark intended. Thus, it is believed that Stark failed to formulate this intention correctly. It is also noted that although some subsequent research [see Teicholz and Ashley (1978); Diekmann et al. (1982)] has made use of other aspects of Stark's linear program, they have not made use of this constraint.

Stark then expressed his "basic model" as having the objective of maximizing the present value (PV) described as follows, subject to the constraints listed and described above:

$$PV = \sum_{j=1}^J \sum_{n=1}^N Q_{nj} (1+r)^{-n} P_j \quad (5)$$

where r =discount rate of interest, in monthly terms.

Table 4. Gates' Example of Payment on Basis of Balanced Bid [Reprinted with Permission from Gates (1967)]

Item number	Description	Unit	Quantity	Unit bid (dollars)	Amount (dollars)
1	Clearing	Lump sum	As necessary	50,000.00	50,000
2	Earth excavation	Cubic yards	70,000	1.50	105,000
3	Rock excavation	Cubic yards	5,000	3.00	15,000
4	Cleaning up	Lump sum	As necessary	50,000.00	50,000
Total bid					220,000

Table 5. Gates' Example of Payment on Basis of Classified Bid [Reprinted with Permission from Gates (1967)]

Item number	Description	Unit	Quantity	Unit bid (dollars)	Amount (dollars)
1	Clearing	Lump sum	As necessary	50,000.00	50,000
2	Earth excavation	Cubic yards	70,000	1.60	112,000
3	Rock excavation	Cubic yards	5,000	1.60	8,000
4	Cleaning up	Lump sum	As necessary	50,000.00	50,000
Total bid					250,000

He described his basic model as being appropriate for those circumstances where the final quantity of each item will exactly match the bill quantity and where the contractor has been exactly able to determine the scheduled rate at which each item will be built. He identified this as a linear programming problem that was capable of being solved within the practical computing constraints of the time.

This basic model thus only addresses the benefit of front-end loading. Stark then developed a further derivative of this model for the purposes of also addressing the benefit of quantity error exploitation and this model (described below) entails the maximization of PV' as follows:

$$PV' = \sum_{j=1}^{J'} \sum_{n=1}^{N'} Q'_{nj} (1+r)^{-n} P_j \quad (6)$$

where J' = estimated final number of items; N' = estimated final number of months; and Q'_{nj} = estimated final quantity of item j expected to be built in month n .

Stark suggested that there are "some circumstances" in which it will be advantageous to rather optimize the present worth of a project's future profit (PV''), expressed as follows:

$$PV'' = \sum_{j=1}^J \sum_{n=1}^N Q_{nj} (1+r)^{-n} (P_j - C_j) \quad (7)$$

where C_j = "known" unit cost of item j .

He did not describe which circumstances should give cause to encourage this alternative approach.

This formulation of a project's profit is simplistic seeing as it does not take into account many factors that determine that a contractor's cash outflow with respect to any item will seldom, if ever, have a simple and continuous linear relationship with the rate at which the item is built [see, for example, Kaka and Price (1991); Kenley (2003)].

Stark illustrated this alternative approach by applying it to his "basic model" [see Eq. (5)]. It would appear, however, that he intended that it should also be applied to his other model [in which he also incorporates quantity error exploitation—see Eq. (6)].

Stark's model was limited to only taking into account the combined benefits of cashflow and quantity estimation errors and it was also limited with respect to its handling of risk. Furthermore, the effect of his model is largely that items are priced at either the upper or lower bounds that the contractor will have imposed as constraints. The model's ultimate result is thus largely dependent on these arbitrary and subjective inputs, which have to be decided without the benefit of any suggested scientific aids.

He did, however, agree with Gates (1967) that there was considerable risk attached to any item price loading strategy.

His suggestion was that contractors should test their pricing models using sensitivity analyses and thereby identify some sense of this risk. This limited approach to risk may be accountable for his concern about having to design his model so as to be practical with reference to the limited (and expensive) computing resources available to contractors at that stage. His model therefore was purely deterministic in nature.

Ashley and Teicholz's Models

While Stark's (1968, 1972, 1974) efforts took into account the combined benefits of cashflow and quantity estimation errors, Ashley and Teicholz's (1977) subsequent research did not build upon this but rather, seemingly independently, proposed a simple linear unbalancing model for the sole purpose of improving a contractor's cashflow. In this initial research, they recognized that some benefit could be derived from "quantity error exploitation" but they concluded that it appeared too difficult to model systematically.

Interestingly, Ashley and Teicholz's model was developed for the express purposes of determining, *prior* to submitting the bid, the extent and manner to which the cashflow of a project might be manipulated by means of item price loading. They envisaged this information would aid in the decision as to whether or not to bid as well as on what amount of bid to submit.

They recommended that, while having to cope with the limited amount of information available in the initial stages of a project, a contractor should identify the following three cashflow curves:

1. **Earnings curve**—representing the value of the contractor's "work-in-place," derived from a schedule of activity for the project priced in terms of the item prices in the Bills of Quantities;
2. **Payments curve**—derived from the earnings curve and adjusted to take into account the retention that is expected to be withheld by the client. This curve will also be "stepped" (as opposed to being a continuous "smooth" curve) to take into account the (typically monthly) intervals between the client's payments as well as the delays (between when work is executed, and then measured and then finally paid for). This curve would therefore represent the contractor's cash inflow; and
3. **Cost curve**—representing the contractor's cash outflow. The cost curve should be derived from the contractor's estimate of his costs, matched together with his schedule of activity, and taking into account any lead or lag between the timing of the activity and the timing of the expected cash outflow. This curve is also to be derived taking into account the analysis of each item's cost, comprising the various types of cost components (labor, materials, plant, subcontractors, and

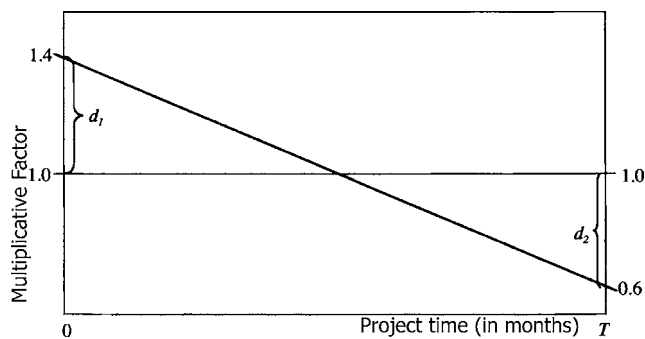


Fig. 1. Linear unbalancing model (Ashley and Teicholz 1977, ASCE)

overheads) and how each of these has a different cash outflow lead or lag. The cost curve may bear little or no resemblance to the earnings or payments curves.

Ashley and Teicholz (1977) suggested that the contractor determine the net cashflow as being the net difference between the payments curve and the cost curve. They also suggested that these differences per period be discounted back to a present value, using two different discounting rates: one representing the contractor's cost of borrowing, being applied to any negative net cashflow amounts; and the other representing the contractor's "corporate rate of return" (being larger than their cost of borrowing) being applied to any positive net cashflow amounts. They conclude that the summation of these periodic (typically monthly) present values would identify the project's "net present worth"—useful as a measure of a project prior to a contractor's commitment to a bid as well as to its constituent item prices.

Ashley and Teicholz (1977) then advocated that a contractor measure the effect of the front-end loading of a project's item prices using their suggestion of a "linear unbalancing model," depicted graphically in Fig. 1.

This linear unbalancing model results in all items scheduled for the start of a project having their prices loaded with the factor d_1 (the "unbalance factor"). Subsequent items would be allocated a corresponding lesser factor, derived from the linear equation linking d_1 at the outset of the project with d_2 at the end of the project. Various values of d_1 can be tested so as to determine the effect on the contractor's cashflow as well as on the project's "net present worth." For each value of d_1 a corresponding, compensatory value of d_2 needs to be determined. Ashley and Teicholz (1977) suggested that d_2 can be found (by means of an iterative search using a computer) such that the overall unbalanced bid retain the same simple cumulative value as the balanced bid, i.e., such that the total "earnings" of the project be kept the same.

Teicholz and Ashley (1978) went on to adopt Stark's (1974) alternative model with some minor differences. They proposed that this represented a more sophisticated "optimal model" that, by comparison to their initial effort (Ashley and Teicholz 1977), now included quantity error exploitation.

The essential differences between their model and Stark's model may be summarized as follows:

1. **Optimization of profit.** Although Stark had mentioned this as a possible alternative to the optimization of a project's revenue, Teicholz and Ashley (1978) give no explanation why they chose only to advocate that a contractor should pursue the maximization of profit.
2. **Probability of execution.** Teicholz and Ashley (1978) de-

scribe the final quantity of an item as the product of two forecasts: the "probability of execution" and the contractor's own estimate of the final quantity.

Their "probability of execution" refers to the contractor's estimate of the chance of any particular item being built. If a contractor believes with certainty that some quantity of a particular item will finally have to be built as part of a project, then they would assign a "probability of execution" for this item of 100%. Similarly, if a contractor were of the opinion that there is only a 50% chance of this item having to be built, then the "probability of execution" would be 50%.

There seems no reason why the latter estimate should not inherently incorporate the former probability. On this basis, the "probability of execution" could be considered a redundant factor. For example, if a contractor were to believe that an item will not finally have to be built, then their estimate of this item's final quantity could simply be expressed as being nil.

3. **Analysis of each item's cost.** Stark's model (in which he seeks to optimize a project's profit, as opposed to its revenue) incorporates a simple forecast of each item's cost. This cost is inherently treated as being directly proportional to the item's quantity.

Teicholz and Ashley incorporate the project's cost estimates somewhat differently. They break the costs down into three categories: fixed, variable, and interest. Their model seeks to optimize the present value of a project's "profit" defined as the revenue less the present value of these three categories of costs. Moreover, they calculate the present value of the "variable costs" of any item on the basis that it is derived from the final quantity that the contractor estimates as appropriate for this item. Thus their model differs slightly from Stark's by recognizing that some items' costs incorporate fixed costs, which will not be affected by any variations in the quantities of these items.

4. **Expressed inclusion of the cost of interest.** Stark's model implicitly incorporates the contractor's cost of interest with the use of discounted cashflows. Teicholz and Ashley instead expressly incorporate the cost of interest and yet they advocate, in addition, the use of further discounting.

Their approach therefore has the error of "double counting" the cost of interest, that is, accounting for it twice when only once is correct. It is also flawed by way of calculating the cost of interest by treating each project in isolation. Their calculation only includes the cost of interest in those months when the project's projected net cumulative cashflow is negative; alternatively they advocate that there is no cost of interest (that is, they only account for interest paid and do not account for interest earned). This approach fails to recognize that the project being analyzed is not one that is in isolation but rather that it inevitably forms part of the contractor's larger "portfolio" of current projects. The cost of interest to the contractor (incorporating knowledge as to whether the contractor has to borrow money or not) cannot therefore be determined without taking the contractor's overall cashflow and level of cash resources into account. One cannot consider a single project in isolation and make the assumption that if the project has a negative cashflow that then the contractor's overall status, inclusive of all his projects, will be such that he will have a negative cashflow. One project's negative cashflow could, for instance, be offset by another project's concurrent positive cashflow. Similarly if a contrac-

tor's status at one stage were such that his overall cashflow were considerably negative, they might be highly dependent, or at very least highly desirous, of as much positive cashflow contribution at that same stage from any new project. A large positive cashflow contribution from the new project would be highly valuable to them relative to a much smaller positive contribution. The former would do very much more to reduce their overall cost of interest incurred than would the latter. If they were to assess the cost of interest at an isolated project level, it therefore makes more sense that they account just the same for the "cost" of interest regardless of whether, at the project level, the net cashflow is positive or negative.

5. **Dropping of Stark's "rate constraints."** Teicholz and Ashley's (1978) approach does not incorporate the need for Stark's "constant of proportionality" (α), which Stark had used as a loading factor to "dial up" the extent of the front-end loading. They have instead been solely reliant on the discounting rate (which is inherent within the method that they used by which to discount the cashflow) by which to achieve the desired degree of pricing preference for early items.
6. **"Desirability index" alternative to the use of linear programming.** Teicholz and Ashley (1978) suggested that the model could be structured as either a linear program (as per Stark's suggestion) or else by using a technique of ranking the items by way of determining a "desirability index" with respect to each item.

They recognized that the effects of their linear programming model (as with Stark's) gave cause for all items, barring one, to be priced at either of the maximum or minimum pricing limits assigned to each item. Only one item would need to be assigned a price that is not its assigned maximum or minimum limit and this item serves to ensure that the summation of the priced component items will equate with the overall bid price.

Teicholz and Ashley (1978) therefore identified that there was no need for linear programming if a contractor could just rank all of a project's items in terms of their "desirability"; in other words, in order of their priority status for being awarded or allocated their maximum-limit price. To start with, this approach allocates all items with their minimum-limit price. Next, they calculate the total for all of the items priced accordingly (using the original quantities as listed within the bills of quantities). If one then deducts this amount from the overall bid amount, one is left with an amount that can serve to be allocated to those items of greatest "desirability." Thus, starting at the item ranked as having the highest desirability index, they advocated that one allocates this item with its maximum-limit price. One then needs to make the corresponding adjustment to what becomes the remaining balance that is still left to be allocated. Moving down and repeating this process to the next ranked item, and then the next, as the balance reduces one eventually will come to the one item that is to be assigned neither its maximum- nor its minimum-limit price. This item's price can then be calculated by using up the remaining balance, and thus the problem (as they defined it) is solved.

Their desirability index (DI_j) for each item is calculated as follows:

$$DI_j = \frac{P_j \sum_{n=1}^N Q'_{nj} (1+r)^{-n}}{Q_j} \quad (8)$$

Their "probability of execution" factor has been left out when translating their formula here for the reason (also explained above) that, in order to be consistent with the formulas throughout this paper, it is considered unnecessary to have both this probability of execution as well as the estimate of the final quantity. Both can be described by way of the latter variable alone.

Notice that their proposed simplified technique no longer incorporates estimates of each item's cost. Each item's price (barring one) is effectively being decided as either the minimum or maximum pricing limit influenced by the following three factors:

- Extent to which the item's quantity is expected to be varied;
- Timing of the item in terms of the project's schedule; and
- Discounting rate.

This approach inherently overcomes the "double counting" of the cost of interest discussed above. In effect, this alternative technique of calculation results in a new model that is substantially different in concept from their linear programming (LP) model. *Inter alia*, this new model seeks to maximize the *revenue* from a project instead of its profit.

This alternative technique also overcomes the need for the contractor to have to incorporate into the model all of each item's estimated fixed and variable costs. This would appear to be both beneficial and problematic. It is problematic because there is merit in identifying all of the fixed costs in a project—those that will arise regardless of variations in the project's quantities. As long as the model identifies and monitors these fixed costs (should there be any), the model is able to ensure some protection against the risk that in the event that an item's quantity is reduced a proportion of its fixed costs will not have been compensated for. By not monitoring any item's costs, this alternative method of computation loses this advantage.

Besides this advantage, there is no other reason why they should need to incorporate any costs into their LP model. There is no logical reason why a contractor's practice of item pricing should be influenced by knowledge of the individual item costs. By implication, it is also of no benefit to the function of item pricing that the contractor should need to have knowledge of the *timing* of these costs. The contractor's costs, as well as the timing of these costs, will arise regardless of the item price combination chosen by the contractor. There is no causal-effect relationship between the benefits from item pricing, and any individual item's cost. Instead, item pricing gives cause for a substantial influence on a contractor's *revenues*, and also of the *timing* of these revenues.

It follows that any effort to incorporate costs, as well as the timing of these costs, in any item price loading model is a wasted effort. The only apparent reason for Teicholz and Ashley (1978) to have incorporated item costs (and their timing) in their model is their express need to estimate the cost of interest. Their model necessitated estimates of each item's cost of interest (and hence, by summation, they would also have estimated the project's total cost of interest). It has, however, been shown above (see the earlier discussion on "double counting" of interest) that, provided that it is to be considered as an item pricing model, their model had, in fact, no logical need for these cost of interest estimates.

Ashley and Teicholz's (1977) original (cashflow planning) model was, however, presented so as to have a dual purpose: it was first said to be intended as a device to determine the attractiveness of a project to a contractor and hence to aid his decision as to whether or not to bid, and of how much to bid. It was also said to be a device to decide appropriate item prices. Given that this model was therefore not solely intended as an item pricing model, there was originally justified cause to have incorporated consideration of each item's cost. In terms of Ashley and Teicholz's (1977) intention of being able to assess a project's "attractiveness," this requires that the contractor be able to quantify its present worth. This, in turn, requires that the contractor be able to forecast a project's net cashflow, and hence there was the need to incorporate the estimates of each item's costs (and also their timing).

However, Teicholz and Ashley's (1978) subsequent model(s) moved away from being intended to serve this broader purpose. These subsequent models were designed with the express sole purpose of identifying what was intended to be "optimal" item prices. When considering this transition in their intent of purpose, it can be argued that their LP model no longer had need to incorporate the elaborate analysis of each item's cost in the manner in which it had previously been done. Their simplified (DI) alternative model (to their LP model) appears to recognize that this observation is true, notwithstanding that they presented it merely as an alternative, quicker, easier method of calculation, and not as an alternative new model.

Overall, Teicholz and Ashley can be described as having largely abandoned their prior "linear unbalancing model" (Ashley and Teicholz 1977) and instead adopted Stark's model. Despite the fact that they improved and replaced their earlier model, largely rendering their previous efforts obsolete, it is their earliest model that is still widely used in recent research [see for instance, Kenley (2003)].

In summary, although their work had the superficial appearance of being substantially original and a significant contribution to this field of knowledge, the effective significance of their work could rather be described as having only identified a quicker and simpler technique of calculation (that can serve as an alternative to the need for linear programming) for the application of Stark's model. It thus did not expand much on what was originally proposed by Stark.

Diekmann, Mayer, and Stark's Model

Diekmann et al. (1982) took Stark's original deterministic model and added a probabilistic formulation to take into account risk. All risks, however, were not considered and the only risk to have been incorporated was the risk that the final item quantities may be different from what the contractor initially anticipated. They ignored all other risks (including the risk of variances in all of the variables used in their model), as if they did not exist. What is, however, of valuable significance as the contribution from this research is that their model facilitated that a contractor could utilize item price loading for the purposes of not only maximizing their profit but also of controlling their risk (albeit in the limited manner that they defined it). They thus accomplished the paradigm leap from all previous research (although they made no reference to the works of Ashley and Teicholz) that had, up until then, simply identified that the practice of item price loading was a risk. They recognized that not only could unbalanced bidding contribute to increased risk, but that it also could be used to

manage and *reduce* risk. Their model provided the framework for a technique that provided a quantifiable means by which the practice of item price loading could be pursued while "balancing" (*their term*) profitability against the measurement and manipulation of a project's risk (no matter that their definition of what constituted risk was limited).

They ignored all risks other than the risk of a variation in item quantities and also only loaded prices in the pursuit of the benefits of cashflow and quantity error exploitation. They therefore ignored the pursuit of increased escalation by way of back-end loading. Their model was also limited by their assumption that the cash inflows and outflows for any item arise simply, without any lead or lag, at the time at which the item is built. Their model thus incorporated the costs of each item but did so without recognizing that the timing of the cash outflows for different items will differ substantially depending on the nature of the item. It can be argued (see the critique of Teicholz and Ashley's model above) that there is no need for this nature of item pricing model to have to incorporate an item's estimated costs nor, by implication, the timing of these costs. Nevertheless, Diekmann et al.'s (1982) model does so although it does so in a manner that may be regarded as simplistic. It does not take into account that different items may have substantially different lead/lag times regarding their associated cash outflows. The differences in these timings are caused by different items comprising differing proportions of different types of constituent costs. To forecast any item's cash outflow should require that one give consideration to the item's cost breakdown into its constituent components which may be of many different types (labor, materials, subcontractors, etc.), each with substantially different delays/advances between when the item is built and when each of these costs have to be paid for.

While their model is derived from Stark's model, they did not implement Stark's "constant of proportionality" (α) constraint. Incidentally, this constraint was also ignored in the subsequent work done by Ashley and Teicholz (1977).

Their model with regard to the expected value of present value (PV) is given below. Their equation has been translated (as with all the equations throughout this paper) to a common format in which the same symbols are used throughout

$$E(PV) = \sum_{j=1}^J A_j \bar{Q}_j (P_j - C_j) - F_0 \quad (9)$$

where

$$A_j = \sum_{n=1}^N (1+r)^{-n} Q_{nj} / Q_j \quad (10)$$

and $E(PV)$ =expected value of the present worth of profit; A_j =factor accounting for the expected timing of item j ; $\bar{Q}_j$ =mean or expected value of Q_j ; and F_0 =present value of the fixed costs of the project.

Thus Diekmann et al. (1982) have presented this model as the probabilistic equivalent of Stark's (1974) deterministic model. They go further to show that, with the assumption that the item quantities are normally distributed, the formulation of the mean and variance of the expected profit are as follows:

$$\mu = \sum_{j=1}^J A_j \bar{Q}_j (P_j - C_j) - F_0 \quad (11)$$

$$\sigma^2 = \sum_{j=1}^J A_j^2 \sigma_j^2 (P_j^o - C_j) - F_0 \quad (12)$$

on the assumption that the covariances between the items are zero where σ_j^2 =variance of Q_j ; and P_j^o =present value of P_j .

If, on the other hand, the variances in the quantities of the items are not mutually independent, then the variance is given as follows:

$$\sigma^2 = \sum_{j=1}^J \sum_{i=1}^J A_i A_j \sigma_{ij} (P_j^o - C_j)(P_i^o - C_i) \quad (13)$$

where σ_{ij} represents the covariance between the variables Q_i and Q_j for $i \neq j$, or the variance of Q_i when $i=j$.

Diekmann et al. (1982) choose to measure risk by way of assessing the probability that the present value of a project's projected profit would not be less than some chosen value V . They go on to show that, for any given value V , by way of this definition, the higher the mean μ the lower the risk, and the lower the variance σ^2 the lower the risk.

On this basis, they choose to combine these in one measure of a project's risk by way of the following objective function (which a contractor is obviously wishing to have maximized)

$$R_o = \mu - k\sigma^2 \quad (14)$$

in which μ is defined by way of Eq. (11) above and σ^2 is defined by way of either Eq. (12) or (13), as appropriate.

This model can be solved as a quadratic program seeing as it comprises an objective function that is of a quadratic format, which is to be solved subject to linear constraints. The linear constraints they use are the same as those identified by Stark, namely those given as Eqs. (1)–(3) above.

They argue that the value V can be intuitively chosen by a contractor to suit their "economic and competitive situation," i.e., that the chosen level of V will depend on their willingness to accept risk.

The contractor's choice of constant value k is even more abstract in concept and more likely to be difficult to decide on. The constant k has the effect of shifting emphasis between the mean μ and the variance σ^2 as being the two sources of risk (as they have chosen to define it). High values of k have the effect of giving preference to minimizing the project's projected profit variance, and hence of avoiding the pursuit of quantity error exploitation, unless the contractor is reasonably certain of the anticipated quantity variation. Very low values of k , e.g., values approaching zero (bearing in mind that k is restricted to being a positive value) will cause Eq. (11) to effectively resemble Eq. (9) in the sense that little to no regard will be made of any item's expected variance.

Diekmann et al. (1982) appear willing to accept that the contractor's choice of the values V and k is difficult and likely to be problematic. In essence they suggest that one experiment with different values (with there being little guidance available on this matter, nor that there are any "right" or "wrong" approaches to this) judging to see what effect different scenarios have on the resultant item prices, as well as on the mean and variance of the expected profit.

Diekmann et al. (1982) provide an example to illustrate the favorable effect of their model. Unfortunately, in this example they chose to withhold imposing any upper limits to the price of each item even though they recommended that in practice these limits are required. Furthermore, in the manner in which the example is captured as a quadratic program, it limits each item's

price to reflect a minimum markup of nil. No explanation is provided as to why some items' prices are not allowed to be less than the corresponding estimated item cost.

Without the imposition of upper price limits, this example therefore enjoys greater freedom by which to pursue its objective. The result is that it does not illustrate at all the problem that arises when one imposes arbitrarily chosen upper and lower pricing limits for each item. What happens in the event of upper and lower pricing limits is that all items barring one will be assigned a price corresponding to either one of these two limits. Thus, in practice, in accordance with their recommendation (and not in accordance with their example) it can be argued that there is no greater influence on this model than these arbitrarily chosen limits. Nevertheless, as with Ashley and Teicholz (1977), Teicholz and Ashley (1978), and Stark (1968, 1972, 1974), the sophistication of this model does not extend to addressing this critical issue.

In conclusion, one could argue that this model constituted a very significant contribution to the science of unbalanced bidding. It was the first model to address the management of both the profitability to be derived from item price loading as well as the risk. However, although Diekmann et al.'s (1982) model is of far greater mathematical sophistication than any model that preceded it, it is believed to be flawed for reason of the following limitations:

1. Consideration that a project's risk is solely related to the risk of variation in each item's quantities (with the treatment of all the other variables being in a deterministic way);
2. Unnecessary complexity of the incorporation into the model of having to forecast a project's cash outflow (for the purposes of optimizing a project's profit rather than its revenue);
3. Contractor having to decide on values (without any scientific aid) for some abstract constants (namely, V and k) which have a critical influence on the model's outcome;
4. Effective use of the constant k by which to "balance" the risk versus return tradeoff, where k is a very arbitrary and abstract measure of this critical decision;
5. Use of two sets of constraints whereby each item's price is simplistically a function of other items' prices, and also that each item's price is constrained by simplistic upper and lower limits (thus it suffers from the same problems as Ashley and Teicholz's model as well as Stark's model); and
6. Limitation to the pursuit of front-end loading and quantity error exploitation only and hence the failure to recognize the benefits of back-end loading.

Cattell's Model

Cattell (1984) identified all three of the potential benefits from item pricing and he wrapped them all up in one comprehensive and cohesive formulation. This early work (Cattell 1984) did not, however, recognize the risks of item pricing [as commented on by Taylor and Bowen (1987)], and it was only in his subsequent work (Taylor and Bowen 1987) that he added to this the recognition that item pricing has a considerable effect on a project's risk. He then formulated a means to measure a project's risk so that one could quantify, comprehensively, both the *risk* and the *return* factors involved in item pricing. He then advocated the use of the quadratic programming techniques commonly used in modern portfolio theory as a novel way by which to manage the decisions regarding the combination of the risks and returns generated by item pricing.

Cattell's (1987) formulation of the PV of a project was described by way of the following equation:

$$PV = \sum_{j=1}^J \sum_{n=1}^N \left(\frac{1}{1+r_j} \right)^n [Q'_{nj}[(1-R_n)P_j - C_j + \gamma_{nj}(fP_j - C_j)] + (1+r_r)^n R_n Q'_{nj} P_j] \quad (15)$$

where r_j =discount rate for item j appropriate to the risk of that item; R_n =proportion retained from payment to the contractor in month n ; γ_{nj} =adjustment for escalation/inflation; f =proportion of the price that is contractually subject to escalation; and r_r =interest rate earned on retained funds.

Thus Cattell (1987) had identified a single, comprehensive formulation by which to determine the contribution to a contractor's profit by way of any single item's price. He reasoned that this formulation comprised numerous variables that are of an uncertain nature. He argued that the various degrees of uncertainty in all of these variables contribute to an uncertainty, or *risk*, in the projected, formulated profit. He suggested that it was possible to estimate the variability in each of these variables and also that it would be necessary to identify any covariances linking any of the risk factors inherent in these variables. He then noted that one could use simple mathematics to combine all of these variances so as to quantify a project's overall risk.

Cattell (1987) suggested that having found the basis by which one could quantify the effect of an item's price on both a project's projected profit as well as its risk, the issue then became one of how to manage this risk versus return tradeoff. He hypothesized that contractors who are rational should have similar motives to any other form of investor with regard to the matters of risk and return, namely, that they should desire as high a return as possible for any given amount of risk, and correspondingly, as little risk as possible for any given amount of return. Cattell found that while it can be expected that different item price combinations will contribute different amounts of both risk and return, the mathematical techniques employed in the field of modern portfolio theory could be applied to facilitate a management of the classic risk versus return tradeoff.

Cattell (1987) concluded that, similar to any analysis of various possible portfolios of equity investments, some combinations of item prices might be regarded as more *efficient* than others. He argued that, as with the assessment of all other nature of investments, a rational contractor should not necessarily prefer a high-return/high-risk item price combination rather than a low-return/low-risk alternative combination. Contractors should, however, be expected to prefer the item price combination that offers them the most return for any given amount of risk to all other combinations that are shown to generate the same amount of risk. Similarly, for any given amount of projected return, they should be expected to try to find the one single item price combination that will generate the least possible risk. Thus, Cattell (1987) argued that rational contractors should try to identify all possible *efficient* item price combinations (that is, all those price combinations offering the most return for the amount of risk involved). He furthermore advocated that the quadratic programming techniques used in modern portfolio theory offer the means by which to do this.

Notice that Cattell's model is also formulated, similarly to Teicholz and Ashley's (1978) model and Diekmann et al.'s (1982) model, so as to maximize a project's profit and not its revenue. It was shown in the critique of Teicholz and Ashley's model above that the maximization of a project's revenue accomplishes

the same effect as when the objective is instead to maximize the project's profit. It is, however, more difficult to seek to maximize a project's profit, for reason that one is then forced to have to incorporate all of a project's costs into such a model. Thus, Cattell's (1984) model suffers from the same problem as Teicholz and Ashley's (1978) model and Diekmann et al.'s (1982) model in this regard, as being unnecessarily complex. Cattell's model is therefore believed to best be simplified and streamlined to leave out all consideration of a project's costs.

Cattell's (1984) model also shares the flaw of the other models by also being dependent, to some degree, on the imposition of maximum and minimum pricing limits for each item. As with the other researchers, Cattell (1984) failed to recognize the significance of these arbitrarily chosen limits and hence did not propose any scientific basis by which to identify the optimum level of these limits.

Tong and Lu's Model

Tong and Lu (1992) developed a method that was focused solely on optimizing the advantage of what they called "error exploitation unbalancing" referred to by Green (1986) as "individual rate loading" and by Cattell (1987) as "loading for anticipated quantity variation orders." In other words, this method ignored the other benefits in the areas of cashflow and escalation.

Tong and Lu considered two alternative models—with the same intended effect: one using LP and the other using a method they called their "minimum–maximum method." Their sole motivation and justification for this second method was due to what they considered the impractical scale of the LP model. They were of the opinion that the Simplex Algorithm would not cope with being able to solve a LP model of the size they were envisaging. However, this runs contrary to current popular computerized usage of the Simplex Algorithm [see Williams (1993)] which manages to be able to solve models comprising well over 1 million constraints fairly quickly (provided that all the variables are continuous and not integral). This incidentally contrasts with mixed integer linear programs (MILP) (where all the variables are not continuous—some variables, by definition, being integers or binary variables) which take far longer to solve. It is typically with MILP (or "MIP"), rather than with pure LP, where the size of the model is likely to be of practical concern. In particular, the concern is typically with regards to the number of integer and binary variables, rather than the number of continuous variables. In the case of Tong and Lu's LP model (in which all the variables are, by implication, continuous), it should not present any practical concern that it will not be quick enough. With this being the case, there should not be a need to consider their "practical" alternative method.

Tong and Lu's (1992) alternative model (their "minimum–maximum model" that they proposed as quicker and easier than their LP model) resembles the "desirability index" model from Teicholz and Ashley (1978). Teicholz and Ashley had likewise proposed their desirability index model as their alternative to their LP model. Tong and Lu (1992), however, make no reference to the work of Teicholz and Ashley (1978).

Their minimum–maximum model therefore did not present any advance on the model of Teicholz and Ashley. Their alternative LP model similarly did not present any practical advance on the modeling technique proposed by Stark.

Discussion

Item pricing models have commonly been designed as linear programming models. Linear programming has the restriction that a model may only have one objective function. This restriction appears to have led to the practice in which early item pricing models sought only to maximize either a project's profit or else its revenue. These models consequently did not incorporate any assessment of a contractor's risk.

Diekmann et al. (1982) chose instead to use quadratic programming and while this technique again has the restriction of only one objective function, they proposed an abstract way (facilitated by being able to use a quadratic equation as the objective function) of pursuing a maximization of the expected profit together with a minimization of the risk. They effectively came to combine these two objectives with the use of a constant (k) for which the contractor has to arbitrarily choose a value. Different values of k have the effect of shifting the emphasis of the objective from the maximization of profit to the minimization of risk.

Cattell (1987) also advocated the use of quadratic programming, in this instance so as to identify the full range of *efficient* item price combinations (representing different risk versus return tradeoffs), rather than seeking to identify only one *optimum* combination. He hypothesized that different rational contractors may have cause to prefer different efficient combinations (depending on their attitude to risk), but none should wish to adopt an inefficient combination in preference to an efficient one.

The only other method that has been found to be employed in all of these models by which to control "the risk" is by means of the use of constraints to impose arbitrarily chosen (upper and lower) limits on each item's price. These limits, rather than any other factor, contribute the greatest effect to both the extent to which a project's profit can be maximized as well as the extent to which a project's risk can be minimized. However, despite the significance of these limits, it is commonly recommended that they should be decided upon without any scientific or mathematical aid. The use of advanced mathematical programming to refine other aspects of item price loading therefore appears somewhat superfluous as long as the most influential factor is, by comparison, left to be handled relatively crudely.

In effect, the only role that the mathematical techniques have to play is to determine the sequence by which to prioritize items that are to be allocated their upper limit price. This sequence thus identifies the one remaining item that will fall between the set of items given their upper-limit prices and the set of items given their lower-limit prices. This one remaining item thus serves to satisfy the constraint by which the unbalanced bid is, in summation, equal to the already determined tender price.

A characteristic shared by all of the models is that they recommend that these limits be arbitrarily decided upon (and regarded thereafter as fixed). In reality, it is highly unlikely that any item should have absolute maximum and minimum prices beyond which a contractor would never submit a price, regardless of the benefits of doing so (whether this be in terms of increased profit and/or decreased risk). In practical terms, these limits may be regarded as more fuzzy than fixed.

Another critical and yet common shortfall of the above-described models relates to their definition of risk. Even in the case of Diekmann et al.'s (1982) model, the only risk factor that was considered and modeled was the risk that the final quantity for some items may be different from that which the

contractor estimates. Cattell's (1987) model is the only one that gives consideration to the risks that are generated from uncertainty in estimating the most appropriate discounting rate, of unit costs, of changes to the project's schedule, as well as of all the other factors that the other models simply treat deterministically.

Conclusion

Scientists who have worked in this field share the view that there is considerable potential by which contractors can use the techniques of unbalanced bidding by which to improve their profitability and/or reduce their risk. Practitioners have applied some of the techniques of item price loading, apparently with significant benefits. Despite this, this practice is believed to be limited to techniques that may be described, in mathematical terms, as fairly simple. Little is known of any application of any of the scientific models that any of the scientists have proposed.

There is considerable potential for further research on this topic, both with regard to the further development of scientific models and with respect to the evaluation of the practical uses of unbalanced bidding. More particularly, it would be interesting to see an evaluation of the application of at least one of the more recent scientific proposals.

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