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Time-cost tradeoff analysis with minimized project financing cost

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ABSTRACT

Time-cost tradeoff analysis allows a contractor to build a project using an optimal schedule that leads to minimum cost. The financing cost can be minimized if the financing decision considers different financing alternatives such as short-term and long-term loans and lines of credit. Financing optimization should be integrated into time-cost tradeoff analysis to minimize total cost and maximize profit. This study proposes an integrated model that performs time-cost tradeoff analysis and financing optimization. A hybrid GALP algorithm is introduced to solve the optimization by combining genetic algorithms (GA) and linear programming (LP). The proposed model is tested using small and large networks: (1) to prove that optimal results are obtained in terms of the financing cost and profit if the proposed integrated model is used, (2) to validate the performance and structure of each model, and (3) to confirm the practicality of the proposed models in large networks.

1. Introduction

Time-cost tradeoff analysis involves accelerated activity durations that are obtained by allocating more resources, and lead to shorter project duration and lower indirect cost at the expense of higher direct cost [1,2]. Thus, schedulers can perform time-cost tradeoff analysis to find the most cost effective project duration. However, the result of time-cost tradeoff analysis does not represent a realistic decision, unless cash availability constraints and financing cost are also considered. The assumption in time-cost tradeoff analysis that unlimited cash is available during the life of a project is not realistic when retainage is withheld by the owner at the time intermediate payments are made, and when intermediate payments are delayed. As a result, the contractor often needs additional funds to avert deficits.

Although several researchers developed different methods for optimizing scheduling problems (e.g., [3–6]), consideration of financing cost was neglected until 2004, when Elazouni and Gab-Allah [7] introduced "finance-based scheduling" in which construction activities are scheduled such that financing cost is considered and cash constraints are satisfied. Although some researchers have studied the finance-based scheduling problem, very few integrated time-cost tradeoff and finance-based scheduling (e.g., [8–12]). A model is proposed in this research that provides an optimal schedule obtained by an optimal use of activity acceleration methods. This schedule results in minimum total cost (including optimal financing cost) and maximum profit. The proposed model differs from all models developed in past studies that integrated time-cost tradeoff and finance-based scheduling in four respects:

(1) Past studies considered only a line of credit without any consideration of different financing alternatives in terms of sources and types of financing, times of cash provisions, interest rates, and repayment options. Also, past studies assumed a predetermined credit limit (a limit that is specified by the lender) for a line of credit. Actually, according to Fathi and Afshar [13], the minimum financing cost and the optimum financing schedule may be different if a credit limit that is predetermined by the lender is considered for each of the several possible financing alternatives. If lenders allow the contractor to specify the required financing, the model needs to calculate the optimal limit that minimizes the financing cost. The proposed model can handle both a predetermined credit limit specified by the lenders and credit limit specified by the contractor.
(2) The models proposed in past studies assume that profit depends not only on direct and indirect costs, but also on financing cost. However, profit can be maximized if financing cost is minimized by using a combination of different financing alternatives. The proposed model calculates the total cost including the minimum financing cost by using different financing alternatives in addition to activity acceleration methods.
(3) Some of the models proposed in past studies use genetic algorithms (GAs) (e.g., [8–11]) and constraint programming (CP) [12] to find an optimum schedule that results in maximum profit, but unfortunately no attempt is made in these models to optimize financing costs by considering different financing alternatives. Several types of financing with different structures, different interest rates, and different times and amounts of borrowing and repayment are

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considered in the proposed model. It follows that the financing cost and the financing variables have to be optimized, in the proposed model, by linear programming (LP). The LP model is formulated for use inside a genetic algorithm (GA) that is used to find an optimal schedule (i.e., using optimal activity acceleration methods) that results in maximum profit. This hybrid GALP algorithm was inspired by the works of researchers who hybridized GAs for more efficient handling of optimization problems [14–17].

(4) None of the past models that used genetic algorithms (GAs) to integrate time-cost tradeoff and finance-based scheduling attempted to eliminate the in-built deficiencies of GAs. In the proposed model, Lee et al.'s [17] recommendations are used to eliminate the deficiencies of GAs by: (1) creating an initial parent chromosome, (2) identifying the optimal values of the GA parameters (i.e., population size, crossover percentage, mutation percentage, and mutation probability), and (3) identifying optimal GA methods (i.e., parent selection and crossover operation methods) that affect the reliability of the results.

The objective of this study is to increase contractor profits by considering time-cost tradeoff while minimizing financing cost. The proposed model provides the contractor with a work schedule that is created by using early activity start times and an optimum financing schedule. The work schedule and optimum financing schedule lead to maximum profit while minimizing the sum of direct, indirect and optimal financing costs between the crash and normal points.

2. Creation of financing flow

To create the financing flow in a project, it is necessary to start out with a work schedule, which is used to create a cash flow forecast. The financing flow (i.e., the timing and the amount of borrowed money, and repayments) depends on this cash flow forecast.

Activity start and finish times should be known if one wants to create a cash flow forecast. The early start, early finish, late start, and late finish times of activities need to be computed. In this study, the critical path algorithm is used to analyze the network and calculate the early and late start and finish times of activities.

A cash flow forecast should be created to find the minimum financing cost. The cash outflow consists of the cost of bonding (C_{bond}), the cost of mobilization (C_{mob}), the direct cost (C_{dc}), fixed overhead cost (C_{foc}), and the variable overhead cost (C_{voc}), whereas the cash inflow is composed of owner payments to the contractor. Since these costs except the cost of bonding (C_{bond}) change when different activity acceleration methods (different duration and cost) are considered for different activities, the cash flow that depends on these costs also change. It should be noted that since the proposed model is to be used after the contract is signed, the contract price (CP) and the cost of bonding (C_{bond}) are well known at the start and remain the same throughout the project.

2.1. The financing flow

Many construction companies go bankrupt due to financial difficulties [18] that arise if the contractor is unable to accurately predict their cash flow, and to anticipate the amount and timing of the required financing in good time [19]. Financial difficulties may result in the contractor being short of cash and sometimes seeking unnecessary credit from lenders.

Borrowing a large amount of funds, especially from one single source of financing may be difficult, and costly for contractors. An alternative to obtaining a single large credit is the use of different financing alternatives that may be obtained from the same or different lenders. In the financing model proposed in this study, different financing alternatives (i.e., short-term and long-term loans, and lines of credit) with different interest rates, credit limits, and different options of taking and repaying money are considered.

In the proposed financing model, a short-term loan can be taken in a series of scheduled loans $STL_{i,\ j,\ k}$ (i = Time of taking the money = Every 1, 3, 6, 9, 12 months) with a predefined schedule of repayments of the principal (j = Time of repaying the principal = After 3, 6, 9, 12 months) and interest (k = Time to pay interest = Every month or at maturity of loan (j)) as long as the total sum of the series of scheduled loans does not exceed a total limit ($L_{STL_{i,\ j}}$) and each loan does not exceed a limit in each period ($L_{STL_{i,\ j}}{}'$).

Unlike short-term loans, which commonly have repayment schedules of a year or less than a year, a long-term loan (LTL) must be repaid in a longer period of one year or more [19,20]. Even though financial institutions are normally willing to lend money for a shorter period of time to better finance their own operations, typically, long-term loans have been used to finance large projects [21]. Therefore, a long-term loan is also considered in the proposed financing model where the loan limit (L_{LTL}) is taken at the beginning of the project and a fixed amount is paid monthly (including both interest and principal) [19]. However, the proposed model also allows the contractor to consider paying off both the principal and compounded interest at the end of the project.

With a line of credit (LC), the contractor can access the specified credit as needed at any time, as long as the sum of the debts (including the interest) does not exceed the maximum limit set in the agreement (L_{LC}) and each withdrawn sum of money from the line of credit in each period does not exceed the specified limit (L_{LC}'). The handling of lines of credit can be more complicated than short-term and long-term loans since in a line of credit, money can be drawn and repaid on an unscheduled basis, without knowing how much money will be borrowed over the life of the line of credit [19,22]. Indeed, the model proposed in this study provides an optimum schedule of borrowed money (i.e., LC_i where i = Time of taking the money = Any time) and an optimum schedule of repaid money (LC_j where j = Time of repaying interest and principal = After 1, 2, …, 12, T_F months where T_F is the final payment made to the contractor) for a line of credit.

It should be noted that credit limits predetermined by a lender can be considered for the alternatives, but so can credit limits specified by the contractor. It is important for the contractor to specify the required credit rather than leaving the lenders guessing as to what credit is required [19]. The proposed model is flexible enough to allow the contractor to accommodate credit limits regardless of whether they are specified by the contractor or the lender. If the credit limit is set by the lender, the model uses the specified credit limit relative to each financing alternative, whereas if the credit limit is not specified by the lender, the proposed model calculates the optimum limits for each financing alternative that result in minimum financing cost. However, regardless of whether the credit limit is specified by the lender or the contractor, the proposed model provides an optimum financing schedule for each financing alternative (i.e., for a short-term loan, for a long-term loan, and for a line of credit).

3. Model optimization

Two types of variables are considered in the proposed model: (1) activity acceleration methods (X_1) and (2) financing variables (X_2). Each acceleration method (m_i) represents the duration and cost of the activity. An activity with a different acceleration method has a different activity duration and cost. Optimization consists of selecting those activity acceleration methods that result in maximum profit after minimum financing cost is calculated for the selected activity acceleration methods. In other words, the proposed financing model considers the impact of different sets of activity acceleration methods to find a schedule that results in maximum profit. The decision variables of activity acceleration methods and financing variables are presented in Eqs. (1) and (2), respectively; where n is the number of activities, $B_{STL_{(i,j)t}}$, B_{LTL}, and B_{LC_t} represent borrowed money, $R_{STL_{(i,j)t}}$, R_{LTL_t} and R_{LC_t} represent repaid money, and $FC_{STL_{(i,j)t}}$, FC_{LTL_t}, and FC_{LC_t} represent financing cost relative to short-term loans, long-term loans, and lines of credit, respectively. As seen in Eq. (2), FC is a function of B and R.

$$X_1 = \{m_i\} \;;\quad i = 1,\ 2,\ \ldots,\ n \tag{1}$$

$$X_2 = \left\{FC_{STL_{(i,j)t}}\left(B_{STL_{(i,j)t}}, R_{STL_{(i,j)t}}\right),\ FC_{LTL_t}(B_{LTL}, R_{LTL_t}),\ FC_{LC_t}(B_{LC_t}, R_{LC_t})\right\};$$
$$i = \text{Time of taking the money} = \text{Every } 1,\ 3,\ 6,\ 9,\ 12 \text{ months};$$
$$j = \text{Time of repaying the principal} = \text{After } 3,\ 6,\ 9,\ 12 \text{ months};$$
$$t = 0,\ 1,\ 2,\ \ldots, T_F \text{ months} \tag{2}$$

3.1. Objective function

Two objective functions are formulated for the proposed model. The main objective is to maximize profit. Finding the maximum profit requires optimized financing for different sets of activity acceleration methods. The second objective is to minimize financing cost. First, the indirect cost (C_{id}) and total cost (C_{tc}) are calculated for the selected activity acceleration methods using Eqs. (3) and (4), respectively. Then, the total financing cost (C_{fin}) is minimized using linear programming (Eq. (5)). Finally, the profit (P) is maximized using genetic algorithms (Eq. (6)), where O_{BS} represents early completion bonus per week, W is the number of weeks that the project is completed ahead of the project duration that is mentioned in the contract, CP is the contract price, and C_{fin} is the result obtained after linear programming optimization (Eq. (5)). Since the proposed model is used after the contract is signed, the cost of bonding (C_{bond}) and the contract price (CP) are known at the outset. In other words, the cost of bonding (C_{bond}) and the contract price (CP) that are used in Eqs. (3) and (6), respectively, are fixed and do not change if different sets of activity acceleration methods are considered. Therefore, it is to the contractor's advantage to keep down the total cost including financing cost if the contractor wants to increase profit. It should also be noted that if the financing cost is not considered or if the cash flow is positive throughout the project, the proposed model would revert to a simple time-cost tradeoff model.

$$\text{Indirect Cost } (C_{id}) = \text{Fixed Overhead Cost } (C_{foc}) + \text{Variable Overhead Cost } (C_{voc}) + \text{Mobilization Cost } (C_{mob}) + \text{Cost of Bonding } (C_{bond}) \tag{3}$$

$$\text{Total Cost } (C_{tc}) = \text{Direct Cost } (C_{dc}) + \text{Indirect Cost } (C_{id}) \tag{4}$$

$$\textit{Minimize}\quad C_{fin} = \sum_{0}^{T_F} \left(FC_{STL_{(i,j)t}} + FC_{LTL_t} + FC_{LC_t}\right) \tag{5}$$

$$\textit{Maximize}\quad P = CP + (O_{BS} \times W) - (C_{tc} + C_{fin}) \tag{6}$$

3.2. Constraints

The first constraint of the cash flow is formulated in Eq. (7). The minimum cumulative net balance of the cash flow (including financing flow) (N_{min}) is set by the contractor if the contractor wishes to have some funds as contingency. The minimum cumulative net balance of the cash flow can be set up as zero if the contractor is comfortable without any contingency funds. The cumulative net balance of the cash flow (including financing flow) (N'_t) is greater than or equal to N_{min} which is never less than zero (Eq. (7)).

$$N'_t \geq N_{min} \;;\quad t = 0,\ 1,\ 2,\ \ldots,\ T_F \text{ months} \tag{7}$$

If the financing alternatives are restricted by lenders, the constraints in Eqs. (8) to (12) are considered. The total amount of borrowed money relative to each alternative ($\sum_0^t B_{STL_{(i,j)t}}$, B_{LTL}, and B_{LC_t}) should not exceed the limit agreed upon by the contractor and the lender ($L_{STL_{i,j}}$, L_{LTL}, and L_{LC}), respectively (see Eqs. (8), (10), and (11)). For the line of credit, it is important to know that the total debt (i.e., sum of all withdrawals plus the financing cost minus all repayments) (left side of Eq. (11)) should not exceed the total credit limit of the line of credit (L_{LC}). Meanwhile, if the amount of money to be borrowed in each period relative to short-term loans and lines of credit ($B_{STL_{(i,j)t}}$, and B_{LC_t}) is restricted, they should not exceed the respective limits of each financing alternative in each period ($L_{STL_{i,j}}'$, and L'_{LC}) (see Eqs. (9) and (12)).

For short-term loans:

$$\sum_{0}^{t} B_{STL_{(i,j)t}} \leq L_{STL_{i,j}}';\quad i = \text{Time of taking the money} = \text{Every } 1,\ 3,\ 6,\ 9,\ 12 \text{ months};$$
$$j = \text{Time of repaying the principal} = \text{After } 3,\ 6,\ 9,\ 12 \text{ months}$$
$$t = 0,\ 1,\ 2,\ \ldots, T_F \text{ months} \tag{8}$$

$$B_{STL_{(i,j)t}} \leq L'_{STL_{i,j}} \;;\quad i = \text{Time of taking the money} = \text{Every } 1,\ 3,\ 6,\ 9,\ 12 \text{ months};$$
$$j = \text{Time of repaying the principal} = \text{After } 3,\ 6,\ 9,\ 12 \text{ months};$$
$$t = 0,\ 1,\ 2,\ \ldots, T_F \text{ months} \tag{9}$$

For a long-term loan:

$$B_{LTL} \leq L_{LTL} \tag{10}$$

For a line of credit:

$$\sum_{0}^{t} (B_{LC_t} + FC_{LC_t} - R_{LC_t}) \leq L_{LC} \;;\quad t = 0,\ 1,\ 2,\ \ldots,\ T_F \text{ months} \tag{11}$$

$$B_{LC_t} \leq L'_{LC};\quad t = 0,\ 1,\ 2,\ \ldots, T_F \text{ months} \tag{12}$$

4. Computational process

First, the proposed financing model that involves several types of financing with different structures, different interest rates, and different times and amounts of borrowing and repayment needs to be modeled, and then solved by using linear programming. Linprog, a math programming solver of MATLAB is used to perform linear programming. Activity acceleration methods are represented by the genes of a chromosome that are used in a genetic algorithm to find maximum profit. Each time a different activity acceleration method (different chromosome) is used, the financing cost is optimized using linear programming, the total cost of the project is calculated for the selected chromosomes, and the main objective (profit) is calculated. Therefore, the proposed model uses a hybrid GALP algorithm to optimize the objective functions under the specified constraints. As shown in Fig. 1, the proposed model is performed in four stages: (1) preparation of input data, (2) identification of optimal values of the GA parameters, (3) identification of optimal GA methods, and (4) execution of the hybrid GALP.

4.1. Input data preparation

In Step 1 (Fig. 1), the schedule information (i.e., activity ID, name, predecessors, and activity acceleration methods (defined by accelerated durations and associated costs)) is read by an automated system using MATLAB from an Excel sheet. In Step 2, two extreme boundaries are set (i.e., the most accelerated project completion time (crash duration) and the normal project completion time (normal duration)). There are several intermediate alternatives between these two boundaries, which are set because they are used for the creation of the financing flow and for the initialization of parent chromosomes. In Step 3, the project cost data and contract terms are sought from the user. The contract price is calculated in Step 4. In Step 5, the financing data are read from the Excel sheet, and in Step 6 the financing flow, constraints, and variables are set up for every project completion time between crash and normal points, and are saved for use by the hybrid GALP algorithm for optimization. Therefore, the hybrid

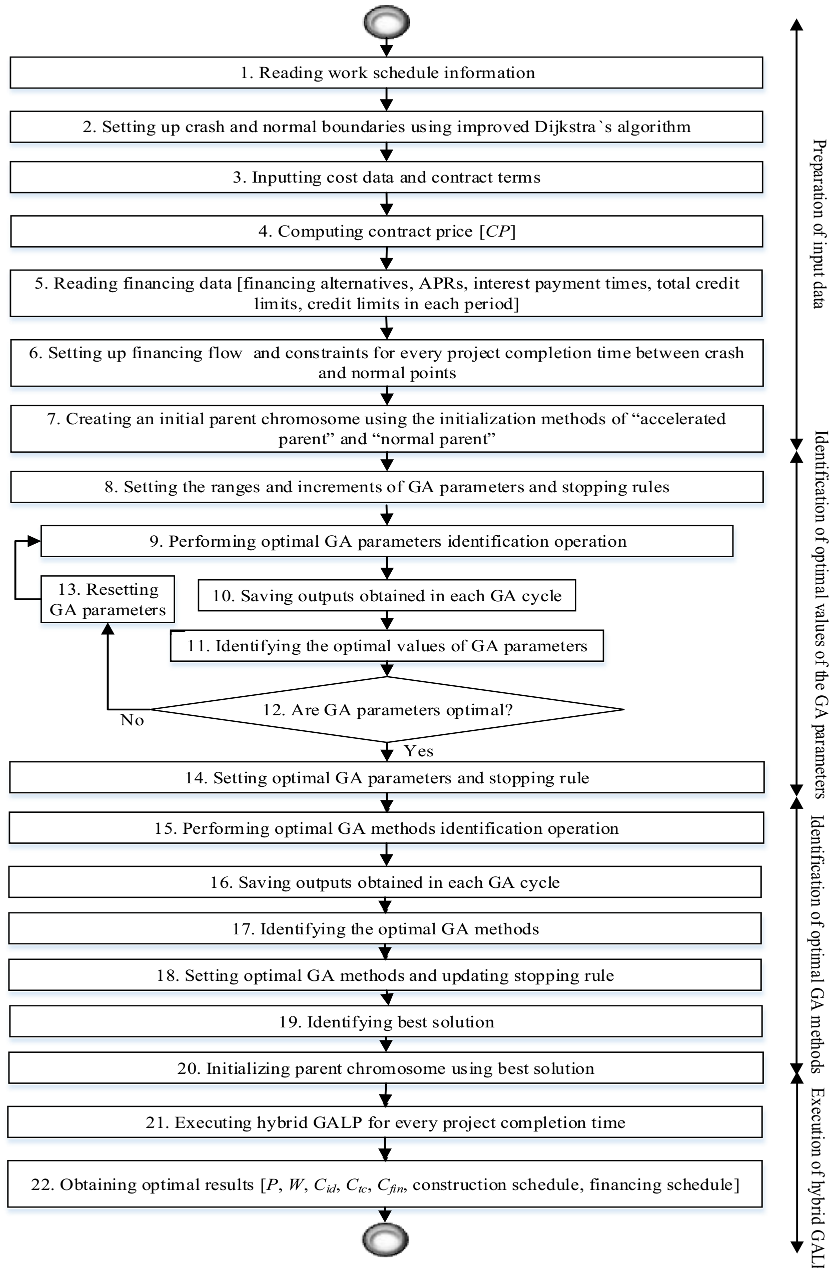

Fig. 1. Algorithm of the proposed model.

GALP algorithm, the related financing flow, constraints, and variables are used to find an optimum schedule for every project completion time between crash and normal points.

In Fig. 2, a sample chromosome representing a set of activity acceleration methods for a project is shown. Generally, the chromosome has a string of N genes. The number of genes in a chromosome is equal to the number of activities (e.g., six genes for a chromosome in Fig. 2 for a schedule that has six activities). The boxes representing activity genes indicate the activity acceleration methods used in those activities. Each activity can be executed by three activity acceleration methods in which each activity acceleration method is associated with a unique activity duration and activity cost. The integer written in the boxes (i.e., 1, 2, or 3) represents the activity acceleration index used in each activity. Activity Acceleration 1 means normal duration and cost, Activity Acceleration 2 means accelerated duration and associated cost, whereas Activity Acceleration 3 represents crash duration and crash cost. It should be noted that financing variables are optimized using linear programming inside the genetic algorithm. Therefore, they are not part of the chromosome.

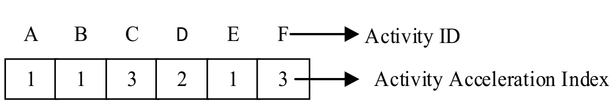

Fig. 2. A sample of encoding chromosome.

In Step 7, an initial parent chromosome is created by adopting an initialization method that was used by Lee et al. [17] rather than initializing all parent chromosomes randomly. Lee et al. [17] proved that their method to generate the initial parent chromosome enhances computational time. In Step 7, two alternative initialization methods are used in the proposed model to test which one is faster. The "accelerated parent" initializes the parent chromosome at an arbitrary point between normal and crash points, whereas "normal parent" initializes the parent chromosome by allocating normal acceleration methods to all activities.

4.2. Identification of the optimal values of the GA parameters

The GA is executed by allocating values to the GA parameters of parent population (N_p), crossover percentage (C_p), mutation percentage (M_p), and mutation probability (M_{pb}). Although past studies selected GA parameters arbitrarily, Lee et al. [17] proposed a method to identify the optimal values of these parameters. Lee et al. [17] proved that their method enhances the reliability and practicality of the GA algorithm. This study adopts Lee et al.'s [17] strategy such that the minimum, maximum, and increment of each GA parameter (i.e., N_p, C_p, M_p, M_{pb}) and stopping rules are set by the user in Step 8. In Step 9, hybrid GALP experiments are conducted using different combinations of the parameters based on controlled experiments to see the changes in the outputs (i.e., computational time and optimal result). All GA parameters (i.e., N_p, C_p, M_p, M_{pb}) alongside the computational time, optimal result, the results obtained after implementing stopping rules, and the number of generations (It_{gen}) that are necessary to reach the optimal solution are saved in Step 10. In Step 11, the optimal values of the GA parameters are identified (i.e., N_p, C_p, M_p, M_{pb}, It_{min}) out of the alternatives identified in Step 10. Similar to Lee et al.'s [17] method, if the optimal result is found at the maximum boundaries of the GA parameters, the upper boundaries of the GA parameters are relaxed and the GA parameters are reset in Step 13 to repeat the process of identifying the optimal values of the GA parameters (i.e., repeating Steps 9–12) using new combinations of the parameters. It should be noted that the reason why the minimum generation (It_{min}) to reach the optimal solution is identified is because it ensures the GA result reaches adequate maturity while reducing the computational time [17]. After identifying the optimal values of the GA parameters, these values along with the stopping rule are set in Step 14 for use in identifying the optimal GA methods.

4.3. Identification of the optimal GA methods

Although Lee et al. [17] fully discussed a model to find the optimal values of the GA parameters, they did not discuss the selection of GA methods (i.e., parent selection method, and crossover operation method) that also affect the computational time and reliability of the results. To create children chromosomes, it is necessary to select two parent chromosomes. The common methods to select the parent chromosomes are “roulette wheel selection” and “tournament selection”.

After two parent chromosomes are selected, it should be decided which crossover operations should be used to create children chromosomes. Current publications use either of the crossover operations “single point crossover”, “double point crossover”, or “uniform crossover”. A fourth crossover operation called “combined crossover” can also be used where all these three operations are adopted based on certain constraints set by the user. In other words, single point, double point, and uniform crossover are used to create pre-set percentages of the children population such that the summation of these percentages equal 100%. No one knows which parent selection method or which crossover operation performs better since the success or failure of these methods and operations depends on the problem (i.e., number of genes, range of search, fitness function, encoding, and GA algorithm) [23]. In the third part of the proposed algorithm presented in Fig. 1, the combinations of selection methods and crossover operations are tested to identify the best selection method and best crossover operation method based on controlled experiments to see what happens to the computational time and the optimal result in Step 15. The outputs are saved in Step 16 and the optimal GA methods are identified in Step 17. In Step 18, the optimal GA methods along with the optimal values of the GA parameters are set and the stopping rule is updated. In Step 19, the best solution, which has been found so far, is identified and is introduced as the initial parent chromosome in Step 20.

4.4. Execution of hybrid GALP

The hybrid GALP algorithm is executed in Step 21 of Fig. 1 to find the optimal solution that undergoes further analysis in Step 22. The detailed description of the hybrid GALP algorithm is provided in Fig. 3. The hybrid GALP algorithm is performed in four phases: (1) initializing the population, (2) evaluating the fitness function, (3) performing the process of evolution, and (4) initializing the next generation population. All the steps described below about the GALP algorithm refer to Fig. 3.

Every time the algorithm runs, the GA parameters and methods are read in Step 1. Using an initialization method, the initial parent chromosome is generated in Step 2 and is evaluated through Steps 3–5. After the initial parent chromosome is evaluated and stored, the parent chromosomes are generated randomly and tested through the fitness function evaluation process in Steps 3–5 until the parent population (N_p) is generated and evaluated in Step 6. When all parent chromosomes have been generated and evaluated, the process of evolution starts in Step 9, based on the chosen parent selection method (i.e., roulette wheel or tournament). Two parents are selected to generate two offsprings in Step 10 using a selected crossover operation method (i.e., single point, double point, uniform, or combined crossover). The crossover process continues until the children population (N_c) that is calculated using Eq. (13) is generated and evaluated.

$$N_c = 2 \times \left\lceil \frac{C_p \times N_p}{2} \right\rceil \tag{13}$$

where C_p = crossover percentage.

When all children chromosomes are generated and evaluated, the mutation process starts in Step 12 by selecting a parent chromosome randomly for mutation. In Step 13, the mutation probability (M_{pb}) is used to specify how many genes should be flipped into something else randomly. The mutation process continues until the mutant population (N_m) that is calculated using Eq. (14) is generated and evaluated.

$$N_m = \lceil M_p \times N_p \rceil \tag{14}$$

where M_p = mutation percentage, and N_p = parent population.

When all mutant chromosomes are generated and evaluated, the population of the next generation is initialized using Steps 14–16. The three populations of parents, children, and mutants are merged ($N_g = N_p + N_c + N_m$) in Step 14 to create a new merged population (N_g) which is sorted in Step 15. The population of the next generation (N_{g+1}) should be equal to the parent population (N_p). The population used in the next generation (N_{g+1}) that is set in Step 16 consists of the top solutions obtained in the previous generation. Then, if the stopping rule is not satisfied in Step 17, crossover and mutation are performed again on the population of the next generation until the stopping rule is met. The best solution is selected in Step 18.

5. Testing the model

The proposed model is tested by using a small network and a large network. The small network is tested for two different financing cases: (1) Case 1: all financing alternatives are considered, as proposed in this research, and (2) Case 2: only a line of credit is considered, as was the case in past research. In addition, when testing the small network, considering all financing alternatives, two different initial parent initialization methods were used to observe the differences in computational time by using the normal initial parent initialization method versus the accelerated initial parent initialization method. Finally, the proposed model is tested using a large network.

5.1. Proposed model vs. models developed in earlier studies

The small network of an example project is shown in Fig. 4. This small network is created by modifying the network adopted by Ali and

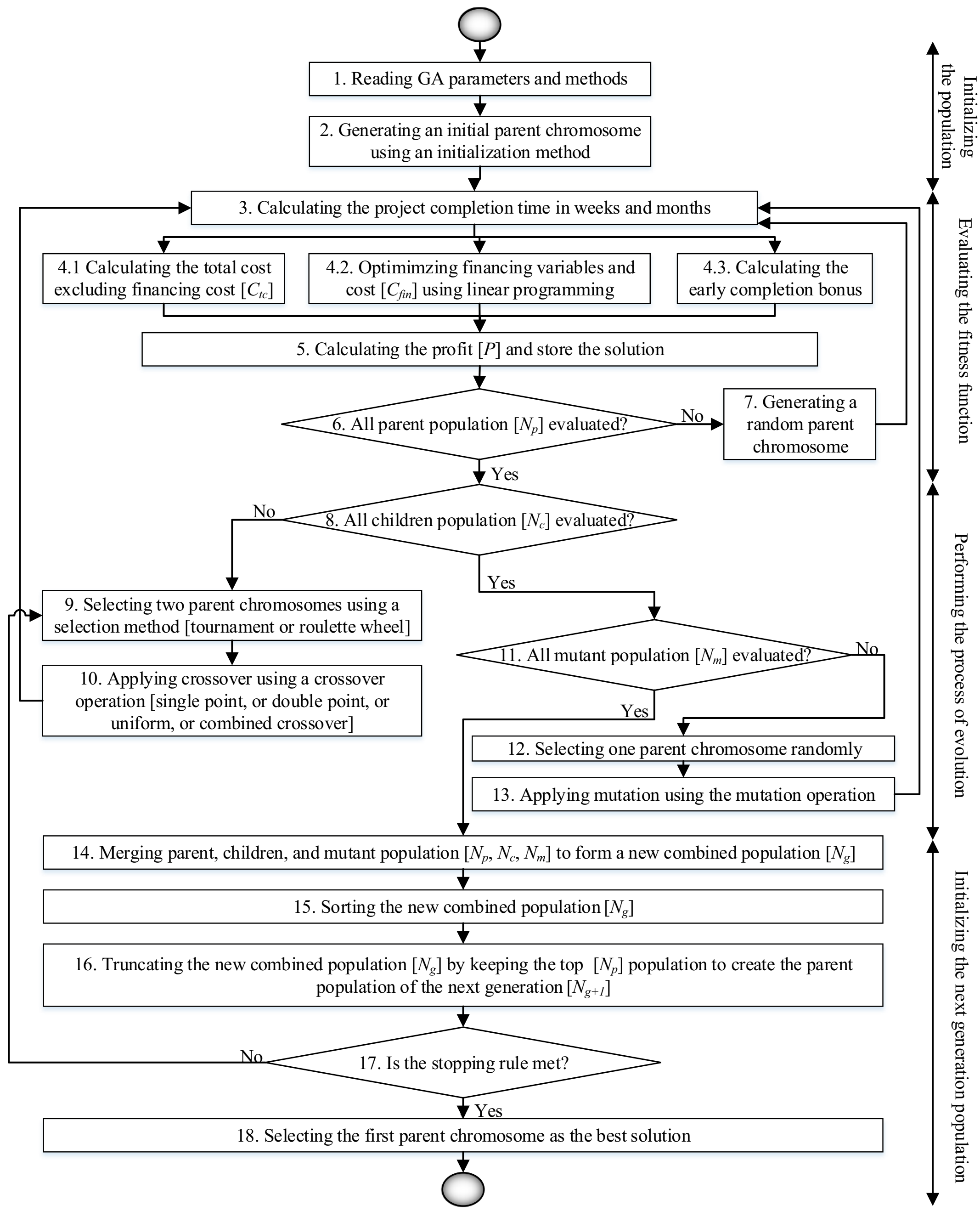

Fig. 3. Hybrid GALP algorithm for the proposed model.

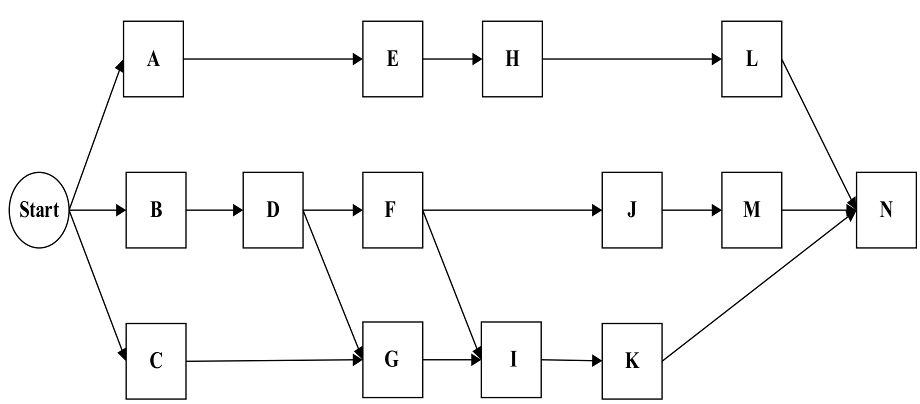

Fig. 4. Small network of the example project.

Table 1
Project schedule data and activity acceleration method inputs.

Activity ID	Activity name	Predecessors			Duration (weeks)			Direct cost ($/week)		
		Predecessor 1	Predecessor 2	Predecessor 3	Acceleration method 1	Acceleration method 2	Acceleration method 3	Acceleration method 1	Acceleration method 2	Acceleration method 3
1	Start	–	–	–	–	–	–	–	–	–
2	A	1	–	–	11	10	9	70,000	90,000	110,000
3	B	1	–	–	4	3	–	40,000	60,000	–
4	C	1	–	–	8	5	4	45,000	70,000	85,000
5	D	3	–	–	3	2	–	50,000	85,000	–
6	E	2	–	–	9	8	–	70,000	90,000	–
7	F	5	–	–	8	7	–	60,000	70,000	–
8	G	4	5	–	10	7	6	55,000	75,000	90,000
9	H	6	–	–	9	8	–	70,000	80,000	–
10	I	7	8	–	11	8	7	60,000	80,000	90,000
11	J	7	–	–	12	11	10	85,000	100,000	115,000
12	K	10	–	–	11	8	7	80,000	110,000	130,000
13	L	9	–	–	9	8	–	60,000	80,000	–
14	M	11	–	–	10	9	8	100,000	115,000	130,000
15	N	12	13	14	12	11	10	70,000	80,000	100,000

Elazouni [8], changing some predecessors and successors and adding some activities. The schedule data and activity acceleration methods of this project, which are shown in Table 1, are read from an Excel sheet by MATLAB 2016a. The cost data and contractual terms of the project, which are shown in Table 2, are sought from the user while running the program inside MATLAB. Then, the financing data are read from another Excel sheet by MATLAB. In Case 1, all proposed financing alternatives are considered. The interest payments of short-term and long-term loans are made monthly, and the optimal timing of the interest payments for a line of credit is determined by the model. In Case 2, only a line of credit is considered, as was the case in all past finance-based scheduling research. Moreover, no borrowing limit is set in either Cases 1 or 2, as the model calculates the optimum amount of borrowed funds for each financing alternative. Furthermore, the annual percentage rates (APRs) which are the annualized rates that consider both compounding and lending fees [19] are the same in both cases. In this study, a line of credit was assigned a 16% APR, a long-term loan 7%, and short-term loans higher APRs than 7% depending on the duration of the loans, ranging between 9% for a twelve-month loan and 22% for a three-month loan to test the proposed model. It should be noted that APRs vary from time to time, from country to country, and even from contractor to contractor and could be different in other cases. However, the proposed model is flexible enough to accommodate as many financing alternatives as available with different APRs that are offered by lenders. A contractor can simply use those alternatives that are available and the APRs of the available financing alternatives, hence reflecting the conditions in any particular project.

The project costs and their calculations are shown in Table 3 where the normal activity durations and normal activity costs are used in all activities.

As seen in Table 4, when tested on the small network (Fig. 4), the model proposed in this research achieves lower financing cost and higher profit in this example compared to past studies that considered single-objective time-cost tradeoff with only one financing alternative (i.e., a line of credit). Even though this outcome is favorable, it should be noted that different APRs can change the result. For example, the proposed model achieves the same result as when only a line of credit is considered if the line of credit has the most favorable APR among the available financing alternatives. The real advantage of the proposed model is to allow a contractor to consider multiple financing alternatives that are available to them, and to run the proposed model based on the APRs they negotiated with the lenders for each financing alternative.

The results for each project completion time are obtained for a set of optimal activity acceleration methods which results in optimal profit for each project completion time. The normal duration of this small network is 52 weeks whereas the crash duration is 43 weeks. In this example, the financing costs in Case 1 (all financing alternatives are considered) are far less than Case 2 (only a line of credit is considered) for all project completion times consistently. It can also be observed that the longer the project completion time, the bigger the difference of financing cost between Cases 1 and 2. Therefore, for a project of longer duration, the importance of using the proposed model can be significant, especially when a long-term loan has the most favorable APR among other financing alternatives. In addition, because of the impact of the financing cost, Table 4 shows that the estimated profit for every project completion time is also higher when the proposed model (Case 1) is used compared to only a line of credit (Case 2) especially when the line of credit has a higher APR than the APR of a long-term loan. As seen in Table 4, the optimal project completion time is 48 weeks, with estimated optimal profits of $1,184,056 for the proposed model and

Table 2
The inputs of cost data and the contractual terms of the project.

Data type	Item	Amount
Cost data	Weekly fixed overhead cost O_{foc}	$100,000/week
	Variable overhead percentage O_{voc} of (C_{dc})	5%
	Mobilization cost percentage O_{mob} of ($C_{dc} + C_{voc}$)	4%
	Profit percentage O_p of ($C_{dc} + C_{foc} + C_{voc} + C_{mob}$)	6%
	Bond premium percentage O_{bond} of ($C_{dc} + C_{foc} + C_{voc} + C_{mob} + P$)	1%
Contract terms	Early completion bonus O_{BS}	$0/week
	Advance payment percentage O_{AP} of contract price (CP)	0%
	Retained percentage R of pay requests	10%
	Number of months between submitting pay requests	1 month
	Lag in paying payment requests (months)	1 month
	Lag to make the final payment and return the retained money (months)	0 month

Table 3
The project costs and contract price calculation using normal activity duration and normal activity cost.

Type of cost/price	Calculation	Amount ($)
Direct cost (C_{dc})	Summation of activities' direct cost using normal activity duration and cost for all activities	8,670,000
Fixed overhead cost (C_{foc})	C_{foc} = 52weeks × \$100, 000/eek	5,200,000
Variable overhead cost (C_{voc})	$C_{voc} = 0.05 \times C_{dc}$ $= 0.05 \times \$8,670,000$	433,500
Mobilization cost (C_{mob})	$C_{mob} = 0.04 \times (C_{dc} + C_{voc})$ $= 0.04 \times (\$8,670,000 + \$433,500)$	364,140
Profit (P)	$P = 0.06 \times (C_{dc} + C_{foc} + C_{voc} + C_{mob})$ $= 0.06 \times (\$8,670,000 + \$5,200,000 + \$433,500 + \$364,140)$	880,058
Cost of bonding (C_{bond})	$C_{bond} = 0.01 \times (C_{dc} + C_{foc} + C_{voc} + C_{mob} + P)$ $= 0.01 \times (\$8,670,000 + \$5,200,000 + \$433,500 + \$364,140 + \$880,058)$	155,477
Contract price (CP)	$CP = (C_{dc} + C_{foc} + C_{voc} + C_{mob} + P + C_{bond})$ $= \$8,670,000 + \$5,200,000 + \$433,500 + \$364,140 + \$880,058 + \$155,477$	15,703,175
Indirect cost (C_{id})	$C_{id} = (C_{foc} + C_{voc} + C_{mob} + C_{bond})$ $= \$5,200,000 + \$433,500 + \$364,140 + \$155,477$	6,153,117
Total cost excluding financing cost (C_{tc})	$C_{tc} = (C_{dc} + C_{id})$ $= \$8,670,000 + \$6,153,117$	14,823,117

Table 4
Optimum results obtained by testing model 1 on small network for financing cases 1 and 2.

Definition of result	Project completion time (weeks)									
	52	51	50	49	48	47	46	45	44	43
Costs excluding financing cost ($)										
Indirect cost (C_{id})	6,153,117	6,051,277	5,946,217	5,847,137	5,750,817	5,660,017	5,571,057	5,482,097	5,394,977	5,306,017
Direct cost (C_{dc})	8,670,000	8,650,000	8,595,000	8,605,000	8,645,000	8,745,000	8,865,000	8,985,000	9,125,000	9,245,000
Total cost (C_{tc})	14,823,117	14,701,277	14,541,217	14,452,137	14,395,817	14,405,017	14,436,057	14,467,097	14,519,977	14,551,017
Financing cost ($)										
Case 1	162,300	153,193	127,461	125,698	123,303	123,465	119,982	122,329	126,033	134,642
Case 2	257,994	246,124	216,637	213,316	207,130	207,176	205,319	208,177	215,481	225,270
Total cost including financing cost ($)										
Case 1	14,985,417	14,854,470	14,668,678	14,577,835	14,519,120	14,528,482	14,556,039	14,589,426	14,646,010	14,685,659
Case 2	15,081,111	14,947,401	14,757,854	14,665,453	14,602,946	14,612,193	14,641,376	14,675,274	14,735,458	14,776,287
Profit ($)										
Case 1	717,759	848,706	1,034,498	1,125,340	1,184,056	1,174,693	1,147,137	1,113,750	1,057,166	1,017,516
Case 2	622,064	755,775	945,322	1,037,722	1,100,229	1,090,983	1,061,799	1,027,902	967,717	926,889

$1,100,229 when only a line of credit is used, a difference of 8%. The optimum schedule that reflects the maximum profit of $1,184,056 in the proposed model (Case 1) is presented in Table 5.

5.2. Analysis of financing results

Generally, the selection of the optimal set of financing alternatives and the amount of money to be borrowed in each alternative depend on

Table 5
Optimum schedule for maximum profit in Case 1.

Activity ID	Activity name	Optimal activity acceleration	Optimal duration (weeks)	Optimal direct cost ($/week)	Optimal start times	Optimal finish times
1	Start	–	–	–	–	–
2	A	Normal	11	70,000	0	11
3	B	Normal	4	40,000	0	4
4	C	Crashed	4	85,000	0	4
5	D	Normal	3	50,000	4	7
6	E	Normal	9	70,000	11	20
7	F	Normal	8	60,000	7	15
8	G	Accelerated	7	75,000	7	14
9	H	Accelerated	8	80,000	20	28
10	I	Crashed	7	90,000	15	22
11	J	Normal	12	85,000	15	27
12	K	Accelerated	8	110,000	22	30
13	L	Normal	9	60,000	28	37
14	M	Normal	10	100,000	27	37
15	N	Accelerated	11	80,000	37	48

Table 6
Cumulative net balance of the cash flow excluding financing flow for normal, optimum, and crash project completion times.

Month	Cumulative net balance of the cash flow excluding financing flow ($)		
	Normal duration (52 weeks)	Optimum duration (48 weeks)	Crash duration (43 weeks)
0	−519,617	−518,567	−543,767
1	−1,570,617	−1,737,567	−1,930,767
2	−1,663,463	−1,426,747	−1,870,269
3	−1,748,304	−1,760,926	−1,996,193
4	−1,745,290	−1,625,217	−1,993,699
5	−1,790,773	−1,755,969	−1,845,111
6	−1,708,198	−1,643,539	−1,717,628
7	−1,625,073	−1,480,575	−1,629,432
8	−1,596,247	−974,178	−985,892
9	−1,455,664	−644,302	−666,224
10	−857,780	−421,047	−761,651
11	−671,533	−522,794	−828,941
12	−909,108	−760,877	1,152,158
13	−1,146,684	1,307,358	–
14	880,058	–	–
Average	−1,208,553	−997,496	−1,201,340

two parameters: (1) the average of the cumulative net balance of the cash flow, and (2) the differences between the APRs based on the duration of keeping the money. The proposed financing model contains three types of financing methods: (1) short-term loans, (2) long-term loans, and (3) lines of credit. Even if the APR of a short-term loan is higher than the APR of a long-term loan, the proposed model usually recommends short-term loans in addition to a long-term loan. The line of credit is also recommended by the proposed model because even though the APR of a line of credit is higher than the APR of a long-term loan, the money borrowed from a line of credit can be paid off in a shorter period, whereas the money borrowed using a long-term loan cannot be repaid in full until the end of the project. The line of credit is desirable where the debt is repaid in a short time, not allowing the interest being compounded. Therefore, whenever the cumulative net balance of the cash flow is negative for several months (sometimes until the end of the project), another financing source that has a lower APR is needed to pay off the debt incurred in the line of credit. As a result, a good choice is the use of a long-term loan that has the lowest APR compared to other financing methods. To justify this analysis, the resulting cumulative net balance of the cash flow excluding financing flow, the schedule of borrowed money, and the schedule of repaid money including interest are presented in Tables 6 to 8, respectively. The cumulative net balance of the cash flow excluding financing flow is presented in Table 6 for crash, optimum, and normal durations. As shown in Table 6, the cumulative net balance of the cash flow is negative until the end of the project for crash, optimum, and normal durations. As a result, as seen in the schedule of borrowed and repaid money in Tables 7 and 8, a long-term loan and a line of credit are optimal alternatives for crash, optimum, and normal durations. By using Table 7, the contractor knows what amount of money should be taken each month in each optimum alternative, and by using Table 8, the contractor knows what amount of money including interest should be repaid each month in each optimum alternative. It should also be noted that the timings and the amounts of borrowing and repaying that are presented in Tables 7 and 8 increase the negotiating power of the contractor with lenders.

A question may arise as to why short-term loans with higher APRs are recommended by the proposed model in the optimal financing solution in addition to a long-term loan with lower APR and a line of credit. There is a difference between the performance of short-term loans and long-term loans. As seen in Table 8, the long-term loan requires a fixed amount of both principal and interest to be paid by the borrower each month until the end of the project, whereas in a short-term loan, just the interest should be paid off monthly and the principal is repaid at the maturity of the short-term loan. Therefore, the negativity of the cumulative net balance of the cash flow increases when only a long-term loan is used since the portion of the principal should be repaid each month. Then, even though the long-term loan has a low APR, more money is needed for the long-term loan to counterbalance the negativity of the cash flow and this additional borrowing results in a higher financing cost. Thus, the optimal decision may include the use of short-term loans in addition to the long-term loan. As seen in the bottom row of Table 6, for crash duration and normal duration, the averages of the cumulative net balance of the cash flow are −$1,201,340 and −$1,208,553, respectively, whereas the optimum duration (48 weeks) results in an average of only −$997,496. Therefore, when the average of the cumulative net balance of the cash flow is negative and large (e.g., −$1,208,553 and −$1,201,340), the schedule of borrowed money (Table 7) shows that a short-term loan is part of the optimal solution, in addition to a long-term loan and a line of credit.

In addition, it should be mentioned that if the average of the cumulative net balance of the cash flow is negative and large (e.g., −$1,208,553 for a normal duration of 52 weeks as seen in Table 6), a larger amount of money should be borrowed using short-term loans (e.g., $364,721 for normal duration as seen in Table 7). On the other hand, if the average of the cumulative net balance of the cash flow is negative but smaller (e.g., −$1,201,340 for a crash duration of 43 weeks as seen in Table 6), a smaller amount of money should be

Table 7
Optimized financing inflow schedule (borrowed money) for normal, optimum, and crash project completion times.

Month	Normal duration (52 weeks)			Optimum duration (48 weeks)		Crash duration (43 weeks)		
	Short-term loan ($STL_{6,\ 12,\ 1}$) ($)	Long-term loan ($)	Line of credit ($)	Long-term loan ($)	Line of credit ($)	Short-term loan ($STL_{6,\ 12,\ 1}$) ($)	Long-term loan ($)	Line of credit ($)
0	364,721	1,981,539	0	2,362,031	0	316,663	2,276,681	0
1	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0
4	0	0	0	0	19,054	0	0	196,556
5	0	0	195,726	0	323,929	0	0	143,759
6	0	0	151,711	0	237,343	0	0	177,430
7	0	0	164,401	0	231,292	0	0	271,087
8	0	0	247,411	0	40,099	0	0	0
9	0	0	239,374	0	0	0	0	0
10	0	0	6789	0	0	0	0	173,860
11	0	0	0	0	256,426	0	0	358,178
12	0	0	752,539	0	554,636	0	0	0
13	0	0	662,300	0	0	–	–	–
14	0	0	0	–	–	–	–	–

Table 8
Optimized financing outflow schedule (repaid money including interest) for normal, optimum, and crash project completion times.

Month	Normal duration (52 weeks)			Optimum duration (48 weeks)		Crash duration (43 weeks)		
	Short-term loan ($STL_{6,\ 12,\ 1}$) ($)	Long-term loan ($)	Line of credit ($)	Long-term loan ($)	Line of credit ($)	Short-term loan ($STL_{6,\ 12,\ 1}$) ($)	Long-term loan ($)	Line of credit ($)
0	0	0	0	0	0	0	0	0
1	2629	147,614	0	188,967	0	2282	196,768	0
2	2629	147,614	0	188,967	0	2282	196,768	0
3	2629	147,614	0	188,967	0	2282	196,768	0
4	2629	147,614	0	188,967	0	2282	196,768	0
5	2629	147,614	0	188,967	4210	2282	196,768	93,296
6	2629	147,614	84,043	188,967	160,806	2282	196,768	105,863
7	2629	147,614	97,284	188,967	205,289	2282	196,768	160,232
8	2629	147,614	125,995	188,967	357,529	2282	196,768	444,490
9	2629	147,614	229,714	188,967	140,908	2282	196,768	0
10	2629	147,614	454,431	188,967	0	2282	196,768	0
11	2629	147,614	36,005	188,967	0	2282	196,768	91,837
12	367,349	147,614	0	188,967	127,587	318,945	196,768	447,870
13	0	147,614	277,111	188,967	695,213	–	–	–
14	0	147,614	1,161,369	–	–	–	–	–

Table 9
Calculation of the cumulative net balance of the cash flow considering an optimal schedule.

Month	Disbursement ($)	Payment ($)	Cumulative net balance of the cash flow excluding financing flow ($)	Borrowed money ($)	Repaid money ($)	Financing cost ($)	Cumulative net balance of financing ($)	Cumulative net balance of the cash flow including financing flow ($)
0	518,567	0	−518,567	2,362,031	0	0	2,362,031	1,843,464
1	1,219,000	0	−1,737,567	0	181,695	7272	2,173,064	435,497
2	993,250	1,304,070	−1,426,747	0	181,695	7272	1,984,097	557,350
3	1,261,000	926,821	−1,760,926	0	181,695	7272	1,795,130	34,204
4	1,224,250	1,359,959	−1,625,217	19,054	181,695	7272	1,625,217	0
5	1,429,000	1,298,248	−1,755,969	323,929	185,853	7324	1,755,969	0
6	1,513,000	1,625,430	−1,643,539	237,343	340,476	9298	1,643,539	0
7	1,570,750	1,733,714	−1,480,575	231,292	383,415	10,841	1,480,575	0
8	1,303,000	1,809,397	−974,178	40,099	531,975	14,521	974,178	0
9	1,072,000	1,401,875	−644,302	0	318,472	11,403	644,302	0
10	820,000	1,043,256	−421,047	0	181,695	7272	455,335	34,289
11	736,000	634,252	−522,794	256,426	181,695	7272	522,794	0
12	736,000	497,918	−760,877	554,636	307,713	8841	760,877	0
13	0	2,068,235	1,307,358	0	866,739	17,441	−123,303	1,184,056
Total	14,395,817	15,703,175	–	4,024,810	4,024,810	123,303	–	–

borrowed using short-term loans $STL_{6,\ 12,\ 1}$ (e.g., $316,663 for crash duration as seen in Table 7). Therefore, if the average of the negativity of the cash flow increases, more money is needed to be borrowed using short-term loans.

Table 9 is presented to explain how the financing flow and the cumulative net balance of the cash flow (including financing flow) are calculated. In Table 9, the disbursements are the actual costs incurred by the contractor when an optimal schedule is considered (optimal costs and durations); whereas the intermediate payments are based on the unit prices of activities, not the actual cost incurred by the contractor. As shown in the last row of Table 9, the contract price (i.e., $15,703,175) does not change when different acceleration methods and an optimal schedule are considered, because the unit prices specified in the original contract are used to calculate the contract price regardless of the contractor's use of different acceleration methods.

In this example, it is interesting to know that even though the contractor keeps more money in his/her possession when the proposed model is used (see Table 9), the financing cost ($123,303) that is paid throughout the project is less compared to when only a line of credit is considered. Therefore, when the proposed model is used, the contractor not only pays less financing cost, but also has more money for use as contingency funds. This result could be different if different financing alternatives with different APRs are considered.

5.3. Analyzing the hybrid GALP algorithm

The small network in Fig. 4 is tested by using two different initialization methods to investigate whether the computational time is shorter than when normal parent initialization method is used. Test 1 was conducted for accelerated parent initialization method, and Test 2 for normal parent initialization method. In addition, a large network of 99 activities is used to confirm the practicality of using the proposed model with large networks. Test 3 was conducted for this large network with the normal parent initialization method to show that when a large number of activities are considered, the proposed model works with no problem. For the large network, the cost and contractual data are identical to the cost and contractual data of the small network, but the configuration of the large network and its schedule information, activity acceleration methods, and the financing data are different. Even though the schedule, the activity acceleration methods, and financing data for the large network are not presented due to lack of space, they are available from the authors upon request.

In addition, the GA parameters are presented in Table 10 for each test. For Tests 1 and 2, 36 combinations exist ($N_p \times C_p \times M_p \times M_{pb} = 2 \times 2 \times 3 \times 3 = 36$) at the beginning of the test. For Test 3, 54 combinations exist ($N_p \times C_p \times M_p \times M_{pb} = 3 \times 2 \times 3 \times 3 = 54$). It should be noted that the GA parameters are relaxed if the optimal result is obtained at the

Table 10
The GA parameters for the small and large networks at the beginning of the test.

GA parameters	Small network (Tests 1 and 2)			Large network (Test 3)		
	Min	Increment	Max	Min	Increment	Max
Parent population (N_p)	30	10	40	150	50	250
Crossover percentage (C_p)	0.6	0.1	0.7	0.7	0.1	0.8
Mutation percentage (M_p)	0.6	0.1	0.8	0.7	0.1	0.9
Mutation probability (M_{pb})	0.12	0.06	0.24	0.08	0.02	0.12

maximum boundaries of the GA parameters. This relaxation occurs in Tests 2 and 3, and increases the number of combinations to 60 and 78 for Tests 2 and 3, respectively.

The proposed model identifies the optimal values of the GA parameters before performing the hybrid GALP algorithm. The optimum GA parameters and methods are obtained and presented in Table 11 to show that depending on different problems, not only are the optimum parameters different for different initialization methods, but also the optimal GA methods are different. It is worthwhile to mention that the "combined" crossover operation that is introduced in this research, is found by the system to be the optimum crossover operation when it is used for the large network in Test 3.

Tests 1 and 2 are performed for different initialization methods to see if the normal parent initialization method results in a shorter computational time. As shown in Table 12, the optimal results of Tests 1 and 2 are identical (i.e., 48 week duration and a profit of $1,184,056), which validates the model by showing that regardless of the initialization method, the model results in the same outcome. In addition, it is observed that the model that starts out with normal parent initialization method performs far better in terms of total computational time compared to starting out with accelerated parent initialization method (i.e., 74,605 s versus 91,475 s). Also, using normal parent initialization method achieves a shorter time per GA cycle (i.e., 956 s versus 1694 s). The normal parent initialization method takes less computational time because the proposed model is designed to calculate the optimum profit for every project completion duration between the normal and the crash durations, starting from the normal duration.

The results in Table 12 show that the model performed well on a large network when the optimal GA parameters and methods are used, even though it took a long time to reach the solution. The model produced an optimal duration of 33 weeks and a profit of $1,839,712. The optimal duration that is obtained for the large network is less than the optimal duration of the small network because the large network had a large number of activities scheduled in parallel, and because the activity durations were shorter.

6. Conclusion

This study proposes a model that shows that considering several financing alternatives has advantages for contractors over using only a line of credit as was the case in all past studies that addressed both the time cost tradeoff problem and the finance-based scheduling problem. These advantages are as follows:

(1) Since borrowing a large amount of funds from one single source of financing is difficult for contractors, the proposed strategy of seeking different financing alternatives from different lenders is a distinct advantage for contractors.
(2) The example presented in this paper shows that the proposed model generates lower financing cost and higher profit than models that use only a line of credit. In this example, the contractor benefits by paying less financing cost and by incurring less total cost, and consequently achieving higher profit. However, it should be noted that the benefit depends on the available financing alternatives and the APRs that are negotiated with lenders. The proposed model allows the contractor to consider the available financing alternatives and their respective APRs to reflect the real conditions in their project.

Table 11
Optimal GA parameters and methods for each test.

GA parameters	Small network		Large network
	Accelerated parent (Test 1)	Normal parent (Test 2)	Normal parent (Test 3)
Parent population (N_p)	30	30	150
Crossover percentage (C_p)	0.6	0.6	0.8
Mutation percentage (M_p)	0.7	0.6	0.8
Mutation probability (M_{pb})	0.18	0.24	0.12
Minimum generation (It_{min})	26	16	33
Selection method	Roulette wheel	Tournament	Roulette wheel
Crossover operation	Single point	Uniform	Combined

Table 12
Validation of models and comparison of computational time between tests.

Test	Process of algorithm	Run time (*sec*)	Number of GA cycle	Total time (sec)	Time per a GA cycle (sec)	Optimal profit ($)	Optimal duration (weeks)
1 (Small network) (Accelerated parent)	Optimal values of GA parameters	62,778	36	91,475	1694	1,184,056	48
	Optimal GA methods	12,075	8				
	Hybrid GALP	16,622	10				
2 (Small network) (Normal parent)	Optimal values of GA parameters	56,906	60	74,605	956	1,184,056	48
	Optimal GA methods	6374	8				
	Hybrid GALP	11,325	10				
3 (Large network) (Normal parent)	Optimal values of GA parameters	558,565	78	764,210	7348	1,839,712	33
	Optimal GA methods	45,444	8				
	Hybrid GALP	160,200	18				

(3) The proposed model considers an optimum combination of different financing alternatives and provides an optimum financing schedule for each project completion time between the normal and crash durations. Presenting an optimum financing schedule to a lender increases the negotiating power of the contractor.
(4) The proposed model can generate optimum financing when the credit limit is pre-determined by the lender. If the credit limit can be specified by the contractor at the start, the proposed model calculates the optimal credit limit and optimal financing, a distinct advantage over past models.

In addition, a hybrid algorithm is introduced in this study that combines genetic algorithms and linear programming (GALP) for the first time in construction financing studies. The proposed hybrid GALP algorithm eliminates the deficiencies of GA algorithms by adopting a controlled experiment operation to adopt a normal parent initialization method, to identify optimal GA parameters and methods, and to introduce a new crossover operation (i.e., combined crossover).

This study has two limitations. First, although changing the start times of activities may reduce the financing cost, this study considers only the early start and early finish times of activities. Second, if the contractor does not have the choice to specify the required credit, the proposed model considers a credit limit that is predetermined by the lender for every financing alternative. If the credit limit predetermined by the lender is not enough to allow the contractor to perform the project between the crash and normal durations, the proposed model does not provide a solution that involves extending the duration of the project. Future research could explore variable activity start times, and extending project duration beyond contract duration.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.autcon.2018.09.009.

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