Optimizing Financing Cost in Construction Projects with Fixed Project Duration

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Abstract: Although a few studies have considered financing cost in the construction project, no research has been conducted to select the optimal financing alternatives out of those offered by financial institutions. This study presents a model that minimizes financing cost by considering different financing alternatives and a work schedule with normal activity durations. The proposed model creates a cash flow forecast based on a work schedule generated by the critical path method (CPM). The output includes optimized schedules of financing inflow (borrowed money) and outflow (repayments of principal and interest). The proposed model has advantages compared to models developed in previous studies because the contractor: (1) pays less financing cost, (2) avoids extending the project duration and avoids liquidated damages, (3) provides an optimal financing schedule, and (4) reduces the risk of ending up with a work schedule composed of more critical activities. The model has been tested in three cases and sensitivity analysis has been performed. **DOI:** 10.1061/(ASCE)CO.1943-7862.0001451.

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Introduction

The contractor's cash inflows and outflows are not balanced in every period during the project, mainly due to retainage, which is the money that the owner withholds to make sure that the project is performed properly by the contractor. Therefore, the contractor will encounter deficits, and may face financial problems. If the contractor insists on using only the money received from the owner, this situation worsens as time goes on and ultimately forces the contractor to completely stop the work. Thus, contractors need to use their capital or borrow money to finance the project. Because using capital or borrowing from a third party comes at a cost, the probable financing cost should be calculated and considered in the preparation of the bid or when the profit is estimated. The estimated total cost and the profit of the project may not be realistic if the financing cost is not considered.

Every year thousands of contractors go bankrupt partly because of the high level of uncertainty in the construction industry. Although many factors could be the cause of business failure, financial and budgetary factors are the most common causes (Arditi et al. 2000). More than 60% of contractor failures have financial causes. The lack of finance causes 77–95% of contractor failures (Russell 1991). The absence of the linkage between financing and project scheduling affects the cash flow and creates nonexecutable schedules that lead contractors to a high rate of failure. Having financing problems not only affects cash flow, but also may influence the interactions among project participants. Conflicts among parties may increase (Harmon 2003), more claims may be filed (Arditi and Pattanakitchamroon 2008), and contract failures may increase (Russell 1991). Thus, not only the integration of financing and scheduling, but also the minimization of financing cost of the project is of vital importance in managing construction projects successfully.

Considering financing costs formally was neglected until 2004 when Elazouni and Gab-Allah introduced *finance-based scheduling*, which integrates the scheduling and financing functions of a construction project and involves the scheduling of construction activities subjected to precedence and financial constraints. The objective of finance-based scheduling is to determine a feasible schedule that minimizes project delay subject to cash constraints while minimizing financing costs (Elazouni 2009; Elazouni and Gab-Allah 2004). If the negative cumulative balance including financing costs exceeds the credit limit, finance-based scheduling modifies the start time of activities by using the activities' total floats (or maybe their durations) and if needed, extends the total project completion time without violating credit limits (Fathi and Afshar 2010). However, all previous studies that considered financing cost have several deficiencies that are discussed in the next section. The objective of this study is not only to eliminate the deficiencies of previous studies, but also to introduce a financing optimization model with respect to different financing alternatives that are available to contractors in terms of sources and types of financing, times of cash provisions, interest rates, and repayment options. The proposed financing optimization model answers three questions in order to maximize the positive cash balance at the end of the project: (1) which financing alternative or combination of financing alternatives should be considered, (2) what amount of money should be taken each month in each alternative, and (3) what amount of money including interest should be repaid each month in each alternative.

Current Finance-Based Scheduling Models

The existing finance-based scheduling models have several deficiencies.

- All pertinent studies conducted so far (i.e., Afshar and Fathi 2009; Alghazi et al. 2012, 2013; Ali and Elazouni 2009;

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Elazouni and Gab-Allah 2004; Elazouni and Metwally 2005, 2007; Elazouni 2009; Elazouni and Abido 2011; Fathi and Afshar 2010; Liu and Wang 2008, 2009, 2010) have considered only one source of financing (i.e., a line of credit) without any consideration of different alternatives of financing in terms of sources and types of financing, times of cash provisions, interest rates, and repayment options.

- Considering just one financing alternative imposes limits on contractors and leads to a financing model that is quite rigid. In other words, the contractor has to borrow a large amount of money from one single lender. This is often difficult and sometimes not convenient for contractors.
- Even though it is desirable for the contractor to use a long-term loan to reduce interest charges and to avoid a large withdrawal from its line of credit (Au and Hendrickson 1986), none of the current finance-based scheduling models have considered long-term loans in their models.
- All previous studies except the ones conducted by Afshar and Fathi (2009) and Fathi and Afshar (2010) considered a predetermined credit limit for only a line of credit, but a financing schedule may be different if one assumes optimum limits for each of several possible financing alternatives (Fathi and Afshar 2010).
- The financing cost is minimum when the volume of work scheduled for execution in the late periods of the project is higher than the volume of work at the early periods because the need for the big portion of the required financing will come up late in the project (Fathi and Afshar 2010). The length of time the borrowed money is kept is shorter when activities are scheduled at their late start time compared to a schedule when activities are scheduled at their early start time. As a result, because the goal of current models is to minimize financing cost by shifting the start time of each activity within its total float seeking an optimum solution (Fathi and Afshar 2010), these models may result in a more critical work schedule.
- If no feasible solution satisfies the cash availability constraints even by shifting the start time of activities, the current schedule is extended to satisfy the constraints (Afshar and Fathi 2009). However, the extension of the project duration could affect the reputation of the contractor and could subject the contractor to liquidated damages.
- None of the finance-based scheduling publications provides a schedule of borrowed money and a schedule of repaid money including interest.
- The contractor may want to include a contingency in the financing schedule to cover unpredicted expenses, or unexpected changes in the cash flow forecast (Hendrickson and Au 2000). A financing model should allow contractors to set the minimum cumulative net balance of the cash flow including the financing flow at the end of each period as contingency funds, but all previous studies disregard this issue.

Normally, bankers or other lenders want to see the cash flow projection and make sure that the cash generated by the operations is enough to pay off the debt along with its interest (Peterson 2013; Strischek 2005). In addition, it is important for a construction company to specify the required financing rather than leaving the lending institution guessing as to what credit is required (Peterson 2013). To solve the problems of predicting and obtaining optimum credit from banks or other lenders, the contractor should first explore several sources and methods of financing, and second, the contractor should present to the lenders a reliable cash flow prediction. Having identified the deficiencies of existing finance-based scheduling models, this study proposes a financing optimization model, not only to eliminate these deficiencies, but also to find the optimum financing cost while providing the schedule of financing without extending the duration of the project and changing the start times of activities.

Methodology and Computational Process

Integrating financing optimization and construction scheduling begins with creating a time-based schedule. It then goes to financing optimization to create the optimal financing schedule. In the proposed method, the durations of activities are taken as deterministic inputs. Even though it is recommended to compare the early and late times of activities to find which schedule satisfies the cash constraints and minimize the financing costs of the project (Russell 1970), as it was stated in the Introduction, this comparison may result in a more critical work schedule for the project. Therefore, this study considers only the *early* start and finish times to minimize the risk of a critical work schedule. This information is used to prepare the cash flow forecast. Then, financing optimization is performed to find the optimum financing cost. Ultimately, the schedules of financing and repayment are generated based on the optimum financing methods and by considering the project schedule. The proposed computational method that leads to optimizing financing is presented in Fig. 1. It is executed by an automated system using *MATLAB*. The computational process is described in the following sections.

Creation of Project Schedule

The critical path method (CPM) is a scheduling technique that is widely used. For creating a cash flow schedule, the start and finish times of activities are needed. CPM calculates the early start, early finish, late start, and late finish of activities. The first step, Step 1 in Fig. 1 is reading the CPM data. The data consists of activity ID, name, predecessors, duration, and direct cost. In Step 2 (Fig. 1), the CPM algorithm is used to analyze the network and calculate the start and finish times of activities.

Creation of Project Cash Flow Forecast

The creation of a cash flow forecast necessitates a separate analysis of the cash outflows (expenditures) and cash inflows (incomes) during the project.

Project Cash Outflows

The total cost of a project (excluding financing cost) is composed of direct cost and indirect cost. The direct cost of the project is the sum of the costs incurred by the contractor to pay for materials, labor, equipment, and subcontractors (Fathi and Afshar 2010; Hinze 2012; Peterson 2013). This study assumes the working week as the time unit of activity duration. In Step 3 (Fig. 1), the costs of the project and the contract bid price are computed. The contractor's weekly direct cost (DC_k) is the sum of the direct costs of the activities undertaken in week k. The total direct cost (DC) is the sum of all weekly costs [Eq. (1)], where W is the project duration expressed in weeks:

$$\mathrm{DC} = \sum_{k=1}^{W} \mathrm{DC}_k; \qquad k = 1, 2, \ldots, W \tag{1}$$

The indirect cost of a project is the sum of fixed overhead, variable overhead (Afshar and Fathi 2009; Fathi and Afshar 2010; Peterson 2013), and mobilization and bonding costs.

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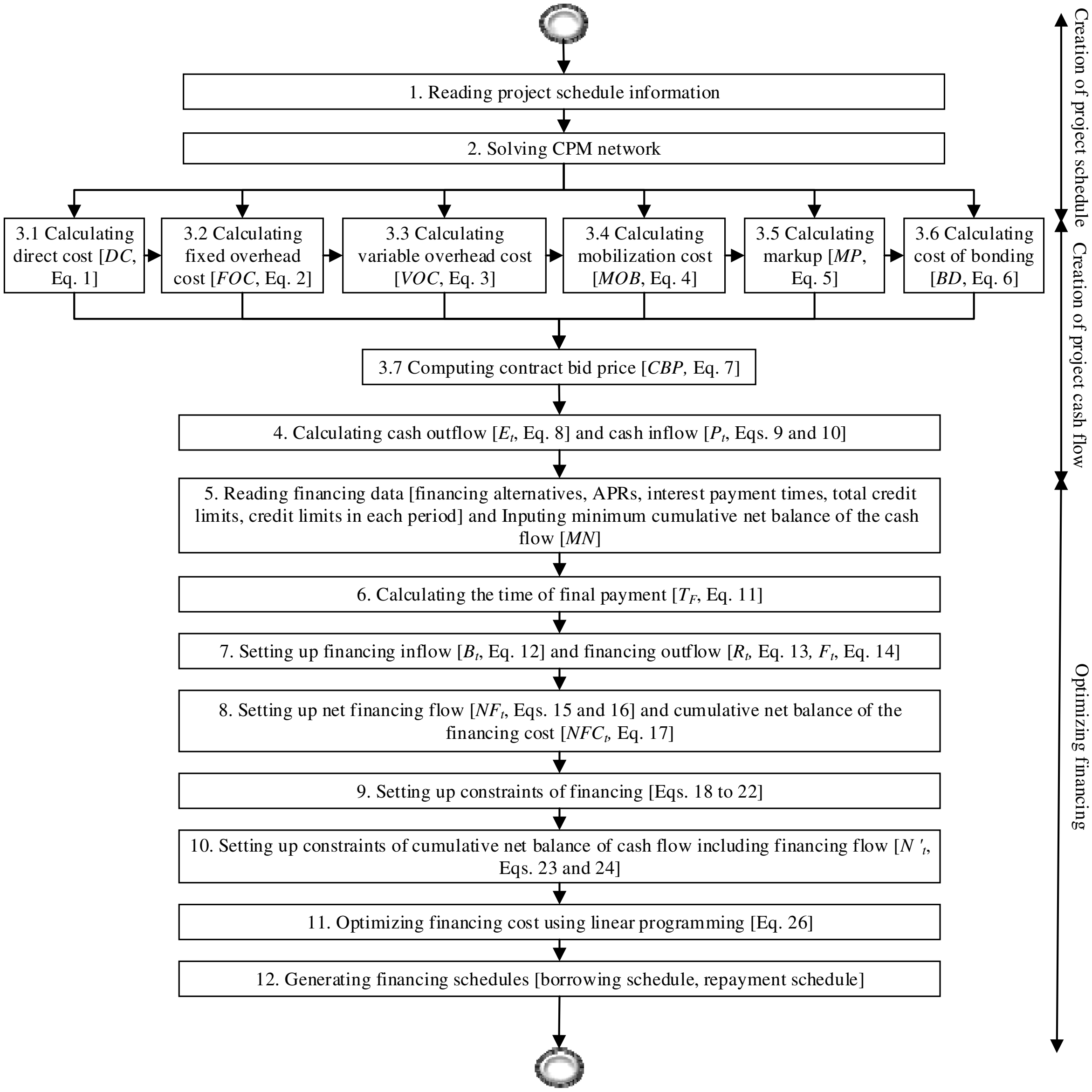

Fig. 1. Model algorithm

Fixed overhead includes administrative expenses (Clough et al. 2005; Peterson 2013). These costs tend to be constant over the duration of a project because, for instance, the rent of the office does not change with the volume of work performed at a construction site (Fathi and Afshar 2010; Peterson 2013). Fixed overhead costs (FOC) should be estimated using historical costs to accurately project the cash flow (Peterson 2013). The FOC of a project is computed by multiplying the FOC per week (O_F) with the number of weeks to complete the project (W) as follows:

$$\mathrm{FOC} = O_F \times W \tag{2}$$

Variable overhead includes utility costs on the job site, personnel scheduling and management costs, taxes, project insurance, and so forth (Afshar and Fathi 2009; Fathi and Afshar 2010; Peterson 2013). Variable overheads tend to vary with the volume of work performed (Fathi and Afshar 2010; Peterson 2013). Several previous studies use a percentage of direct cost to define fixed and variable overhead costs aggregated together (e.g., Ali and Elazouni 2009; Elazouni and Metwally 2007; Liu and Wang 2008). However, based on the definition presented by Afshar and Fathi (2009) and Fathi and Afshar (2010), the variable overhead cost (VOC) is separately computed by multiplying the weekly direct cost (DC_k) with a percentage dictated by the company's historical records (O_V), whereas the FOC is calculated using Eq. (2). The VOC of a project is computed by using Eq. (3):

$$\mathrm{VOC} = \sum_{k=1}^{W} O_V \times \mathrm{DC}_k; \qquad k = 1, 2, \ldots, W \tag{3}$$

Mobilization cost includes those costs that are necessary for the movement and communication of personnel, movement of equipment, supplies, and other facilities required for work to begin on the job site. As per Elazouni and Metwally (2005, 2007), Elazouni (2009), Elazouni and Abido (2011), and Alghazi et al. (2012, 2013), the mobilization cost (MOB) is computed by using Eq. (4) where O_M is the MOB expressed as a percentage of (direct cost + variable overhead cost):

$$\text{MOB} = O_M \times (\text{DC} + \text{VOC}) \tag{4}$$

The markup (MP) of a project is computed by using Eq. (5) where O_{MP} is the MP expressed as a percentage

$$\begin{aligned}\text{MP} = O_{MP} \times (&\text{Direct Cost (DC)} + \text{Fixed Overhead Cost(FOC)} \\ &+ \text{Variable Overhead Cost(VOC)} \\ &+ \text{Mobilization Cost(MOB)})\end{aligned} \tag{5}$$

The cost of bonding (BD) can be calculated only if the total project cost (including direct + indirect costs + markup) is known. Then, the BD is computed by using Eq. (6), where O_B is the premium the contractor pays for the bond expressed as a percentage. The premium rate varies between 0.5 and 2% (Clough et al. 2005).

$$\begin{aligned}\text{BD} = O_B \times (&\text{Direct Cost (DC)} + \text{Fixed Overhead Cost(FOC)} \\ &+ \text{Variable Overhead Cost(VOC)} \\ &+ \text{Mobilization Cost(MOB)} + \text{Markup (MP)})\end{aligned} \tag{6}$$

The contract bid price (CBP) is nothing but the sum of the direct cost (DC), the fixed and variable overheads (FOC + VOC), the mobilization cost (MOB), the cost of bonding (BD), and the markup (MP) [Eq. (7)]. It should be noted that the cost data (O_F, O_V, O_M, O_{MP}, O_B) are input by the user when the cost of project and cash outflow is calculated

$$\text{CBP} = \text{DC} + \text{FOC} + \text{VOC} + \text{MOB} + \text{MP} + \text{BD} \tag{7}$$

In Step 4 (Fig. 1), the cash outflow and cash inflow are calculated. In order to calculate cash outflows and create a cash flow profile, construction disbursement should be calculated for each period separately. The terminology presented by Afshar and Fathi (2009) and Fathi and Afshar (2010) is modified in this study to separate mobilization and bonding costs from other overhead costs. The disbursement at the end of a typical project period t, which is denoted by E_t, is computed by using Eq. (8), where M is the project duration expressed in months. The mobilization and bonding costs occur at the beginning of the project at $t = 0$:

$$E_t = \begin{cases} \text{MOB} + \text{BD} & t = 0 \\ \sum_{k=(4\times t)-3}^{4\times t} (\text{DC}_k + O_F + \text{VOC}_k) & t = 1, 2, \ldots, M \end{cases} \tag{8}$$

Project Cash Inflows

Cash inflows are the owner payments that are due on completion of designated phases of the work. According to Hinze (2012), the payments made by the owner are usually made on a monthly basis. The contractor usually requests payment at the end of each month. Only on rare occasions, do some owners require the contractor to submit pay requests at the end of every 2 months or more. It should also be noted that payment, after deduction of retainage, is made to the contractor after a month or more from the time the payment request is submitted. Moreover, the final payment, which also includes the release of the retainage, is made 1–3 months after the last progress payment is made (Hinze 2012). The retained percentage of pay requests, the frequency with which pay requests are submitted, the owner's lag in responding to payment requests, and the lag to return the retained money with the last payment should be accounted for when calculating project cash inflows. The proposed method allows contractors to reflect the conditions in their particular contract.

Because the contractor regularly submits pay requests every month, t is the time when the pay requests are submitted, and $(t + L_P)$ is the time when the progress payment is made where L_P is the lag in responding to pay requests (in months). The progress payment at the end of each period $(t + L_P)$ is denoted by P_{t+L_P}. Progress payments excluding the last progress payment are computed by using Eq. (9), where R is the retained percentage of pay requests. The ratio of the contract bid price [CBP, Eq. (7)] to total direct cost [DC, Eq. (1)] in Eq. (9) accounts for expenses other than direct cost.

It should be noted that in some contracts, the owner makes an advance payment based on the percentage of the contract price. If there is such a term in a contract, advance payment is included in Eq. (9) where O_{AP} is the advance payment expressed as a percentage of the CBP

$$P_{t+L_P} = \left[(1 - R) \times \frac{\text{CBP}}{\text{DC}} \times \sum_{k=(4\times t)-3}^{4\times t} \text{DC}_k\right] - \frac{(O_{\text{AP}} \times \text{CBP})}{M}; \qquad t = 1, 2, \ldots, M \tag{9}$$

The request for the final payment is submitted after the last progress payment is made. The final payment at $(t + L_R)$, which is denoted by P_{t+L_R}, is calculated by using Eq. (10), where L_R is the lag in making the final payment and returning the retained money (in months). It should be noted that the contract terms (O_{AP}, R, L_P, L_R) are input by the user when the cash inflow is calculated:

$$P_{t+L_R} = R \times \text{CBP}; \qquad t = M + L_P \tag{10}$$

Optimizing Financing Cost

Because it is essential that the agreement between lenders and borrowers reflect exactly how the money will flow, this study aims not only to optimize financing, but also to provide a schedule of financing. To optimize financing cost, there should be multiple alternatives to compare the relative costs of different financing schedules and to ensure sufficient financing to perform and complete the project (Hendrickson and Au 2000). As shown in Table 1, these alternatives are divided into three categories, short-term loans, a long-term loan, and a line of credit. This model is developed such that it considers nearly all possible offers by lenders. According to Peavler (2016), a variety of repayment schedules can be used to pay back loans; however, loans are usually paid back either semiannually or annually. In this study, monthly repayment schedules are proposed for long-term loans, and the repayments of short-term loans are made in 3, 6, 9, or 12 months after the funds are borrowed, therefore covering nearly all possible repayment schedules that can exist in a real project (Table 1).

As far as the timing of borrowing is concerned, as long as the contractor can provide a profitable cash flow forecast including financing costs, the contractor can negotiate a series of loans with lenders (Hendrickson and Au 2000). In this study, short-term loans can be taken either monthly or quarterly (i.e., every 3, 6, 9, or 12 months), whereas the long-term loan is taken once at the beginning of the project (Table 1). However, it is possible that lenders do not offer some or many of these proposed alternatives, or that some of these alternatives do not exist in a specific location. The proposed financing model is flexible enough to handle such situations, such that a contractor can simply disregard those alternatives that are not available, and consider only those alternatives that are offered by lenders.

As per Au and Hendrickson (1986), in this study, in the category of short-term loans, it is assumed that the contractor can use a lump sum in a series of scheduled loans with a predefined schedule of repayments of principal and interest during the project. According to Kramer and Fusaro (2010), lenders are inclined to

Table 1. Alternative Financing Methods

Financing method	Alternative	Time of taking money	Time of repaying principal	Time of repaying interest	Financing code	Total credit limit ($)	Credit limit in each period ($)	Monthly interest rate (%)	Compounded interest rate (%)
Short-term loans	1	Every month (A)	After 3 months	Monthly or after 3 months	A3	$\mathrm{CL_{A3}}$	$\mathrm{CL'_{A3}}$	i_{A3}	$\hat{i}_{\mathrm{A3}} = (1+\mathrm{i}_{\mathrm{A3}})^3 - 1$
	2		After 6 months	Monthly or after 6 months	A6	$\mathrm{CL_{A6}}$	$\mathrm{CL'_{A6}}$	i_{A6}	$\hat{i}_{\mathrm{A6}} = (1+\mathrm{i}_{\mathrm{A6}})^6 - 1$
	3		After 9 months	Monthly or after 9 months	A9	$\mathrm{CL_{A9}}$	$\mathrm{CL'_{A9}}$	i_{A9}	$\hat{i}_{\mathrm{A9}} = (1+i_{\mathrm{A9}})^9 - 1$
	4		After 12 months	Monthly or after 12 months	A12	$\mathrm{CL_{A12}}$	$\mathrm{CL'_{A12}}$	i_{A12}	$\hat{i}_{\mathrm{A12}} = (1+i_{\mathrm{A12}})^{12} - 1$
	5	Every 3 months (B)	After 3 months	Monthly or after 3 months	B3	$\mathrm{CL_{B3}}$	$\mathrm{CL'_{B3}}$	i_{B3}	$\hat{i}_{\mathrm{B3}} = (1+i_{\mathrm{B3}})^3 - 1$
	6		After 6 months	Monthly or after 6 months	B6	$\mathrm{CL_{B6}}$	$\mathrm{CL'_{B6}}$	i_{B6}	$\hat{i}_{\mathrm{B6}} = (1+i_{\mathrm{B6}})^6 - 1$
	7		After 9 months	Monthly or after 9 months	B9	$\mathrm{CL_{B9}}$	$\mathrm{CL'_{B9}}$	i_{B9}	$\hat{i}_{\mathrm{B9}} = (1+i_{\mathrm{B9}})^9 - 1$
	8		After 12 months	Monthly or after 12 months	B12	$\mathrm{CL_{B12}}$	$\mathrm{CL'_{B12}}$	i_{B12}	$\hat{i}_{\mathrm{B12}} = (1+i_{\mathrm{B12}})^{12} - 1$
	9	Every 6 months (C)	After 3 months	Monthly or after 3 months	C3	$\mathrm{CL_{C3}}$	$\mathrm{CL'_{C3}}$	i_{C3}	$\hat{i}_{\mathrm{C3}} = (1+i_{\mathrm{C3}})^3 - 1$
	10		After 6 months	Monthly or after 6 months	C6	$\mathrm{CL_{C6}}$	$\mathrm{CL'_{C6}}$	i_{C6}	$\hat{i}_{\mathrm{C6}} = (1+i_{\mathrm{C6}})^6 - 1$
	11		After 9 months	Monthly or after 9 months	C9	$\mathrm{CL_{C9}}$	$\mathrm{CL'_{C9}}$	i_{C9}	$\hat{i}_{\mathrm{C9}} = (1+i_{\mathrm{C9}})^9 - 1$
	12		After 12 months	Monthly or after 12 months	C12	$\mathrm{CL_{C12}}$	$\mathrm{CL'_{C12}}$	i_{C12}	$\hat{i}_{\mathrm{C12}} = (1+i_{\mathrm{C12}})^{12} - 1$
	13	Every 9 months (D)	After 3 months	Monthly or after 3 months	D3	$\mathrm{CL_{D3}}$	$\mathrm{CL'_{D3}}$	i_{D3}	$\hat{i}_{\mathrm{D3}} = (1+i_{\mathrm{D3}})^3 - 1$
	14		After 6 months	Monthly or after 6 months	D6	$\mathrm{CL_{D6}}$	$\mathrm{CL'_{D6}}$	i_{D6}	$\hat{i}_{\mathrm{D6}} = (1+i_{\mathrm{D6}})^6 - 1$
	15		After 9 months	Monthly or after 9 months	D9	$\mathrm{CL_{D9}}$	$\mathrm{CL'_{D9}}$	i_{D9}	$\hat{i}_{\mathrm{D9}} = (1+i_{\mathrm{D9}})^9 - 1$
	16		After 12 months	Monthly or after 12 months	D12	$\mathrm{CL_{D12}}$	$\mathrm{CL'_{D12}}$	i_{D12}	$\hat{i}_{\mathrm{D12}} = (1+i_{\mathrm{D12}})^{12} - 1$
	17	Every 12 months (E)	After 3 months	Monthly or after 3 months	E3	$\mathrm{CL_{E3}}$	$\mathrm{CL'_{E3}}$	i_{E3}	$\hat{i}_{\mathrm{E3}} = (1+i_{\mathrm{E3}})^3 - 1$
	18		After 6 months	Monthly or after 6 months	E6	$\mathrm{CL_{E6}}$	$\mathrm{CL'_{E6}}$	i_{E6}	$\hat{i}_{\mathrm{E6}} = (1+i_{\mathrm{E6}})^6 - 1$
	19		After 9 months	Monthly or after 9 months	E9	$\mathrm{CL_{E9}}$	$\mathrm{CL'_{E9}}$	i_{E9}	$\hat{i}_{\mathrm{E9}} = (1+i_{\mathrm{E9}})^9 - 1$
	20		After 12 months	Monthly or after 12 months	E12	$\mathrm{CL_{E12}}$	$\mathrm{CL'_{E12}}$	i_{E12}	$\hat{i}_{\mathrm{E12}} = (1+i_{\mathrm{E12}})^{12} - 1$
Long-term loan	21	Beginning of project	Monthly or end of project	Monthly or end of project	LP	—	$\mathrm{CL'_{LP}}$	i_{LP}	$\hat{i}_{\mathrm{LP}} = (1+i_{\mathrm{LP}})^{T_F} - 1$
Line of credit	22	Any time	After 1 month	Monthly or after 1 month	LC1	$\mathrm{CL_{LC}}$	$\mathrm{CL'_{LC}}$	i_{LC}	i_{LC}
			After 2 months	Monthly or after 2 months	LC2				$\hat{i}_{\mathrm{LC2}} = (1+i_{\mathrm{LC}})^2 - 1$
			After 3 months	Monthly or after 3 months	LC3				$\hat{i}_{\mathrm{LC3}} = (1+i_{\mathrm{LC}})^3 - 1$
			After 4 months	Monthly or after 4 months	LC4				$\hat{i}_{\mathrm{LC4}} = (1+i_{\mathrm{LC}})^4 - 1$
			After 5 months	Monthly or after 5 months	LC5				$\hat{i}_{\mathrm{LC5}} = (1+i_{\mathrm{LC}})^5 - 1$
			After 6 months	Monthly or after 6 months	LC6				$\hat{i}_{\mathrm{LC6}} = (1+i_{\mathrm{LC}})^6 - 1$
			After 7 months	Monthly or after 7 months	LC7				$\hat{i}_{\mathrm{LC7}} = (1+i_{\mathrm{LC}})^7 - 1$
			After 8 months	Monthly or after 6 months	LC8				$\hat{i}_{\mathrm{LC8}} = (1+i_{\mathrm{LC}})^8 - 1$
			After 9 months	Monthly or after 9 months	LC9				$\hat{i}_{\mathrm{LC9}} = (1+i_{\mathrm{LC}})^9 - 1$
			After 10 months	Monthly or after 10 months	LC10				$\hat{i}_{\mathrm{LC10}} = (1+i_{\mathrm{LC}})^{10} - 1$
			After 11 months	Monthly or after 11 months	LC11				$\hat{i}_{\mathrm{LC11}} = (1+i_{\mathrm{LC}})^{11} - 1$
			After 12 months	Monthly after 12 months	LC12				$\hat{i}_{\mathrm{LC12}} = (1+i_{\mathrm{LC}})^{12} - 1$
			End of project	Monthly or end of project	LCT_F[a]				$\hat{i}_{\mathrm{LCT}_F} = (1+i_{\mathrm{LC}})^{T_F - t^b} - 1$

[a] T_F is when the final payment is made and retainage is returned.
[b] t is the current time.

lend money for short periods of time to better finance their own operations. The prearranged schedule of financing for a series of loans can provide lenders with a way to treat such loans as short-term loans. As a result, bank capacity is enhanced by lending without charging high premiums for the principal they would have to retain for long-term loans (Kramer and Fusaro 2010). For example, for Alternative 1 in Table 1 (i.e., A3), the contractor can take a series of loans every month and pay off the principal of each loan after 3 months. In addition, the interest can also be paid monthly or compounded over 3 months. The reason why this type of model is considered for short-term loans, is that some lenders allow both principal and interest to be paid off at the maturity of the loan, whereas others require the contractor to pay the interest at regular intervals with the principal being paid at the end of the period (Peterson 2013). It should be noted that the total sum of a series of short-term loans does not exceed an upper limit negotiated with the lenders (e.g., for Alternative 1 in Table 1, the sum of the loans is within $\mathrm{CL_{A3}}$).

In terms of the long-term loan, only a lump sum can be taken at the beginning of the project. In addition, most lenders require the contractor to pay back a fixed amount monthly to cover both interest and principal, whereas others allow the contractor to pay off both the principal and compounded interest at the end of the project (Peterson 2013). Therefore, the long-term loan is not taken as a series of loans.

The line of credit is considered as one account with different modes. Contrary to a financing alternative that has a different rate of interest and a different credit limit (as discussed in the preceding two paragraphs), in the line of credit there is one interest rate and one credit limit. The following reasons are discussed as to why different modes (i.e., different principal and interest payment times) are considered in the line of credit.

- A line of credit does not need to have a prearranged schedule if one considers just a line of credit as the only source of financing. The purpose of the different modes that are considered for the line of credit in Table 1 is to compare the modes not only with loan alternatives but also with each other to find the optimum timing and the amount of withdrawals and repayments including the interest. Regardless of the modes used (i.e., LC1 to LC12, and LCT_F; where T_F is the time when the final payment is made and retainage is returned, in months), the money borrowed plus interest must never violate the credit limit (i.e., $\mathrm{CL_{LC}}$) at any time during the project. In addition, there is a difference between line of credit and short-term loans; the total credit withdrawn from a line of credit can be reused as long as the contractor pays back his/her debts periodically, whereas a short-term loan is fixed and cannot be reused.
- Despite the complexity and unscheduled nature of a line of credit, this research provides a schedule for managing a line of credit. Considering all proposed modes for the line of credit in this research, financing optimization results in a schedule that shows what amount of money should be withdrawn each month from the line of credit account and what amount of money including its interest should be paid back each month to the line of credit account.

In addition to a total credit limit, some lenders may impose a limitation on the amount of a loan or the amount of withdrawal from a line of credit in each period. For example, considering Alternative 1 in Table 1, as long as the sum of a series of loans for Alternative 1 does not violate the total limit (i.e., $\mathrm{CL_{A3}}$), each loan in each period should not violate the limit in each period either (i.e., $\mathrm{CL'_{A3}}$). This might also be the case for a line of credit. In addition to the total amount withdrawn from the line of credit including its interest not violating the total credit limit during the project (i.e., $\mathrm{CL_{LC}}$ in Alternative 22, Table 1), some lenders may require the contractor to withdraw not more than a percentage of the total credit limit in each period (i.e., $\mathrm{CL'_{LC}}$ in Alternative 22, Table 1). As a result, if there is a credit limit in each period, the proposed model is also able to consider that.

Typically, the interest rate is stated in terms of annual percentage rate (APR) (Hendrickson and Au 2000). Effective APR is an annualized rate that considers both compounding and lending fees (i.e., the fees to cover the cost of setting up a loan or a line of credit) (Peterson 2013; Prakash 2015). In this study, it is assumed that a lender quotes the contractor the effective APR, and then the monthly interest rate is calculated.

The financing data (i.e., financing alternatives, APRs, interest payment times, total credit limits, and credit limits in each period) are input by the user in Step 5 (Fig. 1) to find the optimum combination of financing [i.e., schedule of financing inflow (borrowed money) and financing outflow (repaid money including interest)]. This model also allows the contractor to input a contingency by choosing a minimum cumulative net balance of the cash flow (MN). If the contractor is comfortable operating without any contingency funds, the MN is set to zero, but if the contractor wishes to have some funds at the end of each period as a contingency, the MN is set up as such.

Project Financing Flow Creation

For better representation, the time when the final payment is made and retainage is returned, which is denoted by T_F, is computed by using Eq. (11) in Step 6 (Fig. 1). To create the financing flow, the borrowed money (financing inflows), the repaid money (financing outflows), and financing costs (financing outflows) are set up in Step 7 (Fig. 1) by using Eqs. (12)–(14), where B_t, R_t, and F_t represent the borrowed money, the repaid money, and the financing cost, respectively, at the end of period t. In Eqs. (12)–(14), B_{ab_t}, B_{LP_t}, and $B_{\mathrm{LC}c_t}$ represent borrowed money; R_{ab_t}, R_{LP_t}, and $R_{\mathrm{LC}c_t}$ represent repaid money; and F_{ab_t}, F_{LP_t}, and $F_{\mathrm{LC}c_t}$ represent financing cost relative to each financing alternative, where a = time of taking money for short-term loans every A (1 month), B (3 months), C (6 months), D (9 months), and E (12 months), b = time of repaying the principal for short-term loans after 3, 6, 9, and 12 months, and c = time of repaying the principal for a line of credit after 1, 2, 3, …, 12, T_F months

$$T_F = M + L_P + L_R \tag{11}$$

$$B_t = B_{ab_t} + B_{\mathrm{LP}_t} + B_{\mathrm{LC}c_t}; \qquad t = 0, 1, 2, \ldots, T_F \tag{12}$$

$$R_t = R_{ab_t} + R_{\mathrm{LP}_t} + R_{\mathrm{LC}c_t}; \qquad t = 0, 1, 2, \ldots, T_F \tag{13}$$

$$F_t = F_{ab_t} + F_{\mathrm{LP}_t} + F_{\mathrm{LC}c_t}; \qquad t = 0, 1, 2, \ldots, T_F \tag{14}$$

The net financing outflow, which is the total money to be repaid including financing cost at the end of period t, is denoted by TR_t and computed by Eq. (15). In Step 8 (Fig. 1), the net financing flow, which is denoted by NF_t, is computed by Eq. (16):

$$TR_t = R_t + F_t; \qquad t = 0, 1, 2, \ldots, T_F \tag{15}$$

$$NF_t = B_t - TR_t; \qquad t = 0, 1, 2, \ldots, T_F \tag{16}$$

Also, the cumulative net balance of the financing cost (i.e., interest), which is denoted by NFC_t, is computed in Step 8 by using Eq. (17), where NFC_{t-1} is the cumulative net financing costs from period 0 to period $(t-1)$:

$$NFC_t = \begin{cases} F_t & t = 0 \\ NFC_{t-1} + F_t & t = 1, 2, \ldots, T_F \end{cases} \tag{17}$$

Constraints of Financing

In Step 9 (Fig. 1), the constraints of financing are set up using Eqs. (18)–(22). The total amount of borrowed money relative to each short-term loan alternative (T_{ab}) should not exceed the total limit for each short-term loan alternative (CL_{ab}) [Eq. (18)]

$$\begin{aligned} & T_{ab} = \sum_{t=0}^{T_F} B_{ab_t} \le \mathrm{CL}_{ab} \\ & a = \text{time of taking the money for short-term loans every } A\ (1 \text{ month}), B(3 \text{ months}), C(6 \text{ months}), D(9 \text{ months}), E(12 \text{ months}); \\ & b = \text{time of repaying the principal for short-term loans after } 3, 6, 9, 12 \text{ months}; \\ & t = 0, 1, 2, \ldots, T_F \end{aligned} \tag{18}$$

The amount of borrowed money at the end of each period (t) for short-term loans (B_{ab_t}) and a long-term loan (B_{LP_t}) should not exceed the limits of short-term loans (CL'_{ab}) and a long-term loan ($\mathrm{CL}'_{\mathrm{LP}}$) in each period ($t$), respectively [Eqs. (19) and (20)]:

$$\begin{aligned} & B_{ab_t} \le \mathrm{CL}'_{ab}; \\ & a = \text{time of taking the money for short-term loans every } A(1 \text{ month}), B(3 \text{ months}), C(6 \text{ months}), D(9 \text{ months}), E(12 \text{ months}); \\ & b = \text{time of repaying the principal for short-term loans after } 3, 6, 9, 12 \text{ months}; \\ & t = 0, 1, 2, \ldots, T_F \end{aligned} \tag{19}$$

$$B_{\mathrm{LP}_t} \le \mathrm{CL}'_{\mathrm{LP}}; \qquad t = 0 \tag{20}$$

The required credit at the end of each period (C_{LC_t}) should not exceed the total credit limit of line of credit ($\mathrm{CL}_{\mathrm{LC}}$) [Eq. (21)]. The withdrawal from the line of credit at the end of each period (TB_{LC_t}) also should not exceed the credit limit in each period ($\mathrm{CL}'_{\mathrm{LC}}$) [Eq. (22)]:

$$C_{\mathrm{LC}_t} \le \mathrm{CL}_{\mathrm{LC}}; \qquad t = 0, 1, 2, \ldots, T_F \tag{21}$$

$$TB_{LC_t} \le \mathrm{CL}'_{\mathrm{LC}}; \qquad t = 0, 1, 2, \ldots, T_F \tag{22}$$

Constraints of the Cash Flow

In Step 10 (Fig. 1), the cumulative net balance of the cash flow (including financing flow), in period t, which is denoted by N'_t, is computed by using Eq. (23)

$$N'_t = \begin{cases} P_t - E_t + NF_t; & t = 0 \\ N'_{t-1} + P_t - E_t + NF_t; & t = 1, 2, \ldots, T_F \end{cases} \tag{23}$$

The minimum cumulative net balance of the cash flow (including financing flow) is denoted by MN and the constraint N'_t is provided in Eq. (24)

$$N'_t \ge \mathrm{MN}; \qquad t = 1, 2, \ldots, T_F \tag{24}$$

The contractor can choose whether the MN (including financing flow) at the end of each period is zero or larger than zero. A balance greater than zero will of course generate more financing cost than a balance of zero.

Objective Function

In Step 11 (Fig. 1), the model is optimized by using linear programming. The financing variables (X) are considered in the objective function and are presented in Eq. (25)

$$\begin{aligned} & X = \{(B_{ab_t}, B_{\mathrm{LP}_t}, B_{\mathrm{LC}c_t}), (R_{ab_t}, R_{\mathrm{LP}_t}, R_{\mathrm{LC}c_t}), (F_{ab_t}, F_{\mathrm{LP}_t}, F_{\mathrm{LC}c_t})\}; \\ & a = \text{time of taking the money for short-term loans every } A(1 \text{ month}), B(3 \text{ months}), C(6 \text{ months}), D(9 \text{ months}), E(12 \text{ months}); \\ & b = \text{time of repaying the principal for short-term loans after } 3, 6, 9, 12 \text{ months}; \\ & c = \text{time of repaying the principal for a line of credit after } 1, 2, \ldots, 12, T_F; \\ & t = 0, 1, 2, \ldots, T_F \end{aligned} \tag{25}$$

The objective function is the minimization of the total financing cost (FC), and is represented by Eq. (25). The monthly financing schedule of borrowed money and the monthly financing repayment schedule including interest are provided in Step 12 (Fig. 1). The constraints of the linear programming model are represented by Eqs. (18)–(24)

$$\text{Minimize } FC = \sum_{0}^{T_F} F_t \tag{26}$$

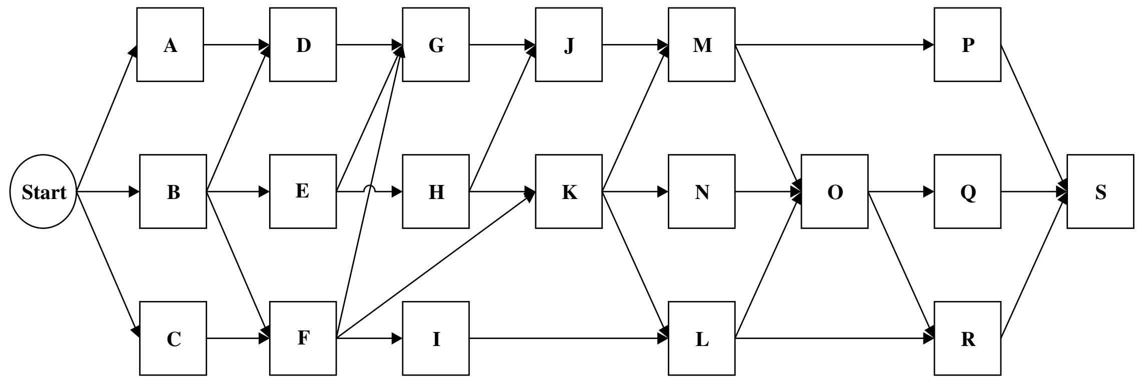

Fig. 2. Network of example project

Table 2. Project Schedule Data Inputs and Model Output of CPM Calculations

Activity identifier	Activity name	Predecessors 1	Predecessors 2	Predecessors 3	Duration (weeks)	Direct cost ($/week)	CPM calculations: Early start	Early finish	Late start	Late finish	Total float	Start time	Finish time
1	Start	—	—	—	0	0	0	0	0	0	0	0	0
2	A	1	—	—	10	50,000	0	10	3	13	3	0	10
3	B	1	—	—	6	100,000	0	6	5	11	5	0	6
4	C	1	—	—	12	60,000	0	12	0	12	0	0	12
5	D	2	3	—	4	60,000	10	14	13	17	3	10	14
6	E	3	—	—	3	40,000	6	9	11	14	5	6	9
7	F	3	4	—	5	90,000	12	17	12	17	0	12	17
8	G	5	6	7	5	20,000	17	22	17	22	0	17	22
9	H	6	—	—	8	30,000	9	17	14	22	5	9	17
10	I	7	—	—	7	60,000	17	24	20	27	3	17	24
11	J	8	9	—	6	90,000	22	28	22	28	0	22	28
12	K	7	9	—	5	100,000	17	22	22	27	5	17	22
13	L	10	12	—	9	60,000	24	33	27	36	3	24	33
14	M	11	12	—	8	40,000	28	36	28	36	0	28	36
15	N	12	—	—	5	70,000	22	27	31	36	9	22	27
16	O	13	14	15	4	70,000	36	40	36	40	0	36	40
17	P	14	—	—	6	90,000	36	42	41	47	5	36	42
18	Q	16	—	—	7	60,000	40	47	40	47	0	40	47
19	R	13	16	—	5	50,000	40	45	42	47	2	40	45
20	S	17	18	19	7	60,000	47	54	47	54	0	47	54

Table 3. Inputs of Cost Data and the Contractual Terms of the Project

Data type	Item	Amount
Cost data	Weekly fixed overhead cost O_F	$25,000/week
	Variable overhead percentage O_V of (DC)	10%
	Mobilization cost percentage O_M of (DC + VOC)	5%
	Markup percentage O_{MP} of (DC + FOC + VOC + MOB)	6%
	Bond premium percentage O_B of (DC + FOC + VOC + MOB + MP)	1%
Contract terms	Advance payment percentage O_{AP} of contract bid price	0%
	Retained percentage R of pay requests	10%
	Number of months between submitting pay requests	1 month
	Lag in paying payment requests L_P (months)	1 month
	Lag to make the final payment and return the retained money L_R (months)	0 month

Testing the Model

The network of an example project is shown in Fig. 2. This network is a modified version of the network used by Ali and Elazouni (2009), including additional activities and modified predecessors and successors. It is used to demonstrate the procedure explained in the preceding sections and carry out the application of the financing model. The schedule data of this project are read from an Excel sheet by *MATLAB*. The schedule data are used to calculate the CPM by using the CPM algorithm. After running the model, the schedule data and output values of CPM calculations are displayed in tabular format in *MATLAB* (Table 2). For this project, which lasts 54 weeks (i.e., rounded up to 14 months), the cost data and contract terms dictate the information presented in Table 3. The inputs used in this research were inspired from the inputs that were considered in previous studies (e.g., Alghazi et al. 2013; Fathi and Afshar 2010). After the user enters the cost data and contract terms as shown in Table 3, the costs of the project and the contract bid price are calculated in *MATLAB* (Table 4). Then, the project cash flow

Table 4. Project Costs and Contract Price Calculation

Type of cost/price	Calculation	Amount ($)
Direct cost (DC)	Summation of activities' direct cost	7,550,000
Fixed overhead cost (FOC)	$\text{FOC} = O_F \times W = 54$ weeks $\times$ \$25,000/week	1,350,000
Variable overhead cost (VOC)	$\text{VOC} = O_V \times \sum_{k=1}^{W} \text{DC}_k = 0.1 \times \$7,550,000$	755,000
Mobilization cost (MOB)	$\text{MOB} = O_M \times (\text{DC} + \text{VOC}) = 0.05 \times (\$7,550,000 + \$755,000)$	415,250
Markup (MP)	$\text{MP} = O_{MP} \times (\text{DC} + \text{FOC} + \text{VOC} + \text{MOB}) = 0.06 \times (\$7,550,000 + \$1,350,000 + \$755,000 + \$415,250)$	604,215
Cost of bonding (BD)	$\text{BD} = O_B \times (\text{DC} + \text{FOC} + \text{VOC} + \text{MOB} + \text{MP}) = 0.01 \times (\$7,550,000 + \$1,350,000 + \$755,000 + \$415,250 + \$604,215)$	106,745
Contract price (CP)	$\text{CBP} = \text{DC} + \text{FOC} + \text{VOC} + \text{MOB} + \text{MP} + \text{BD} = \$7,550,000 + \$1,350,000 + \$755,000 + \$415,250 + \$604,215 + \$106,745$	10,781,210
Bid price factor (BPF)	$\text{BPF} = \text{CBP}/\text{DC} = \dfrac{\$10,781,210}{\$7,550,000}$	1.4280

forecast is computed using Eqs. (8)–(10) to be integrated with the project financing flow in the next process.

In terms of calculating financing cost and proving that this model is far more optimal than all past finance-based scheduling models, three different financing cases are considered. In Case 1, all proposed financing alternatives are considered. The interest payments of short-term and long-term loans are made monthly, whereas the optimum timing of interest payments of a line of credit is determined by the model (i.e., interest can be paid monthly or compounded over more than a month). In addition, no credit limit is considered for Case 1. In Case 2, the long-term loan is removed and only short-term loan alternatives and a line of credit are considered. In addition, in Case 2, a total credit limit of \$100,000 is considered for the line of credit whereas no limit is considered for short-term loans. The interest payment times for both short-term loans and line of credit are the same as in Case 1. In Case 3, as it was the case in all past finance-based scheduling research, just a line of credit with no consideration of total credit limit is considered. The effective APRs are the same in all three cases and are based on market conditions. APRs of 2–28% for loans and 5–40% for lines of credit are offered by traditional banks and alternative lenders (Mclntyre 2017; White 2017). If the loan is obtained for a longer duration, a lower APR is offered by lenders (Au and Hendrickson 1986). Both financial and nonfinancial factors in addition to the contractor's ability to repay the borrowed money with interest are considered by lenders when they specify APRs for borrowers. If the requirements are met by the contractor, lenders are content with a low APR, whereas if the contractor does not satisfy the requirements fully, higher APRs will be sought by lenders. In this study, a long-term loan is assigned a 7% APR; short-term loans are assigned higher APRs than 7% depending on the duration of the loans, ranging between 8% for a 12-month loan to 23% for a 3-month loan; and a line of credit is assigned a 15% APR. These APRs represent the average market conditions and this assignment does not favor one alternative over the other.

Analysis of the Results

As shown in Fig. 3, Case 1, where all financing alternatives are considered, results in the minimum cumulative financing cost at the end of the project, whereas Cases 2 and 3 result in more financing costs. It should be noted that if one adopts Case 3 as it was the case in all earlier studies, one incurs far more financing costs compared to Case 1, which is proposed in this research (Fig. 3 and Table 5). As shown in Table 5, if the contractor uses Case 1, the total financing cost of the project amounts to \$120,202, whereas by adopting Cases 2 and 3, the total financing cost of the project amounts to \$147,314 and \$201,038, respectively. It should be noted that in Cases 1 and 2, not only is the financing cost less than in Case 3, but also the contractor can avoid a large withdrawal on its line of credit. For example, as shown in Table 5, in Case 1, the required limit for the line of credit is \$274,457, whereas this amount is as high as \$1,552,110 in Case 3. It should also be noted that lenders may charge a higher interest rate when the required credit limit is very high, which makes Case 3 less desirable because of a higher financing cost. Meanwhile, in Case 1, the profit amounts to \$484,013, whereas in Cases 2 and 3, the profit amounts to \$456,901 and \$403,177, respectively (Table 5). Therefore, the contractor achieves far more profit compared to previous finance-based scheduling models by using the financing model proposed in this study. The proposed method could be particularly advantageous in a long-term project with a large contract price.

The total borrowed money (B_t), total repaid money (R_t), total financing cost (F_t), net financing flow (NF_t), cumulative net financing cost (NFC_t), and cumulative cash flow (including financing flow) (N_t') are used to generate the schedule of borrowed money and the schedule of repaid money including interest in Tables 6 and 7, respectively. Interestingly, as shown in Table 6, in Case 2 where the long-term loan is removed and a total credit limit is considered for the line of credit, not only are the optimum alternatives different compared to Case 1, but the schedule of borrowed money

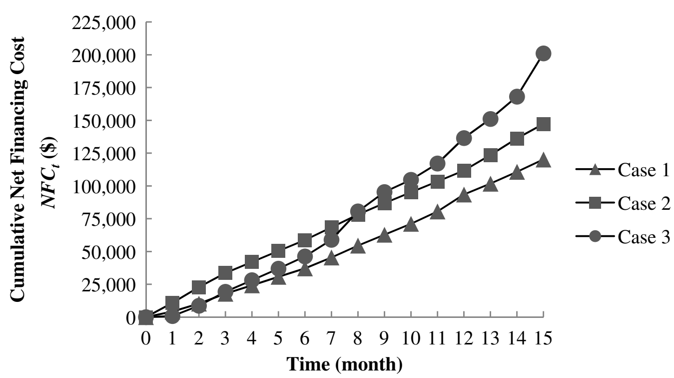

Fig. 3. Cumulative net financing cost NFC_t in three financing cases

Table 5. Summary of Project Financial Parameters Considering Three Cases

Financial parameters	Case 1	Case 2	Case 3
Total financing cost of the project (\$)	120,202	147,314	201,038
Profit of the project (\$)	484,013	456,901	403,177
Required limit for line of credit (\$)	274,457	100,000	1,552,110

Table 6. Model Output of Optimized Financing Inflow Schedule (Borrowed Money) for Three Cases

	Case 1				Case 2				Case 3
Month	Short-term loan (B12) ($)	Short-term loan (C9) ($)	Long-term loan (LP) ($)	Line of credit (LC) ($)	Short-term loan (B12) ($)	Short-term loan (C9) ($)	Short-term loan (E3) ($)	Line of credit (LC) ($)	Line of credit (LC) ($)
0	0	0	1,377,532	0	1,236,592	52,559	167,668	0	521,995
1	0	0	0	264,507	0	0	0	100,000	1,093,258
2	0	0	0	72,594	0	0	0	0	383,785
3	351,126	0	0	0	0	0	0	0	301,449
4	0	0	0	0	0	0	0	0	305,139
5	0	0	0	0	0	0	0	0	325,653
6	0	230,086	0	0	0	40,158	0	100,000	399,449
7	0	0	0	62,996	0	0	0	56,927	320,242
8	0	0	0	0	0	0	0	0	98,444
9	0	0	0	0	0	0	0	0	102,495
10	0	0	0	212,345	0	0	0	0	682,928
11	0	0	0	158,599	0	0	0	0	327,901
12	0	0	0	0	0	0	738,351	96,470	188,688
13	0	0	0	91,437	0	0	0	71,893	304,689
14	0	0	0	37,106	0	0	0	0	284,285
15	0	0	0	0	0	0	0	0	0
Total borrowed money or withdrawal	351,126	230,086	1,377,532	899,582	1,236,592	92,717	906,019	425,290	5,640,399

Table 7. Model Output of Optimized Financing Outflow Schedule (Repaid Money Including Interest) for Three Cases

	Case 1				Case 2				Case 3
Month	Short-term loan (B12) ($)	Short-term loan (C9) ($)	Long-term loan (LP) ($)	Line of credit (LC) ($)	Short-term loan (B12) ($)	Short-term loan (C9) ($)	Short-term loan (E3) ($)	Line of credit (LC) ($)	Line of credit (LC) ($)
0	0	0	0	0	0	0	0	0	0
1	0	0	96,044	0	7,956	419	2,448	0	69,258
2	0	0	96,044	164,098	7,956	419	2,448	101,171	571,334
3	0	0	96,044	178,164	7,956	419	170,116	0	477,777
4	2,259	0	96,044	0	7,956	419	0	0	303,394
5	2,259	0	96,044	0	7,956	419	0	0	270,759
6	2,259	0	96,044	0	7,956	419	0	0	267,666
7	2,259	1,835	96,044	0	7,956	739	0	85,373	357,384
8	2,259	1,835	96,044	63,734	7,956	739	0	73,577	599,437
9	2,259	1,835	96,044	0	7,956	53,298	0	0	274,566
10	2,259	1,835	96,044	0	7,956	320	0	0	161,667
11	2,259	1,835	96,044	98,974	7,956	320	0	0	368,414
12	2,259	1,835	96,044	277,672	1,244,548	320	0	0	566,498
13	2,259	1,835	96,044	0	0	320	10,781	69,493	313,390
14	2,259	1,835	96,044	63,410	0	320	10,781	101,171	410,728
15	353,385	231,921	96,044	66,979	0	40,479	749,132	0	829,165

is different as well. In Case 1, the optimum financing alternatives are B12 (money can be taken every 3 months and should be repaid after 12 months), C9 (money can be taken every 6 months and should be repaid after 9 months), LP (money should be taken at the beginning of the project and be repaid at the end of the project), and LC (money can be withdrawn at any time and be repaid at any time as well), whereas in Case 2, the optimum financing alternatives are B12, C9, E3 (money can be taken every 12 months and should be repaid after 3 months), and LC. Even though three of the four financing alternatives are the same in Cases 1 and 2 (i.e., B12, C9, and LC) and the interest rates are the same, the timings and the amounts of borrowing and repaying in Cases 1 and 2 are different from each other (Tables 6 and 7).

As shown in Table 6, in Case 1, in contrast to the belief that the long-term loan is not a good idea for temporary needs, the big portion of the borrowed money should be taken as a long-term loan (i.e., $1,377,532 out of 2,858,326). The reason is that if the money is withdrawn from a line of credit and is not repaid for a period of time, the interest is compounded. However, the interest of a long-term loan is paid monthly. Therefore, when the money withdrawn from the line of credit and its interest cause the cash flow to be negative, there is a need for another source such as a long-term loan to prevent the financing cost from being compounded. For example, in Case 1, all withdrawn money from the line of credit is repaid after 1 or 2 months because the long-term loan is used to repay the debt as early as possible.

In addition, as shown in Table 6, considering Case 1, the short-term loans of 12- and 9-month periods (i.e., alternatives B12 and C9) taken on months 3 and 6, are better for the borrower than a long-term loan. It is true that the interest rates of alternatives

B12 and C9 are higher than the interest rate of the long-term loan, but because of the shorter duration and the condition of the cash flow in those months, alternatives B12 and C9 are more advantageous than taking a long-term loan.

Another aim of this paper is to provide a schedule for managing a line of credit despite the complexity and unscheduled nature of a line of credit. As shown in Tables 6 and 7, this objective is achieved. In addition, the proposed financing model can also provide the optimum timing of the repayment of the interest and withdrawn money from the line of credit (Table 7).

Sensitivity Analysis and Validation of the Model

In addition to the fact that changes in financing input data change the total financing cost of the project, changes in the contract terms also can change the financing cost. As shown in Table 8, a sensitivity analysis of the best and worse financing cases (i.e., Cases 1 and 3) is performed considering changes in the contract terms about the percentage of the monthly payment retained by the owner (R), the percentage of the contract bid price advanced by the owner at the start of the project (O_{AP}), and the lag in owner payments (L_P) (months) to see the effects of contract terms on total financing cost.

As shown in Table 8, in sensitivity analysis Number 1, the owner not only pays the contractor without any lag (i.e., $L_P = 0$), but also does not withhold any retainage (i.e., $R = 0$). There is a financing cost even in this case, because the contractor does not receive any advance payment at the beginning of the project where the contractor incurs mobilization and bonding costs. In sensitivity analysis Numbers 5, 6, 8, and 9, where the owner makes an advance payment and does not withhold any retainage, the contractor still has to pay the financing cost because of the lag in owner payments.

In sensitivity analysis Numbers 4, 7, 13, and 16, no financing cost is incurred when the owner makes an advance payment, and makes the intermediate payments without any delay. However, in sensitivity analysis Numbers 13 and 16, even if the owner withholds retainage, the total financing cost amounts to 0 because the markup covers the amount of retainage in each period. On the other hand, in sensitivity analysis Number 25, the owner again makes an advance payment in addition to paying the intermediate payments without any delay, but the retained percentage is more than profit; therefore, the contractor pays financing cost.

The importance of retainage can be observed in sensitivity analysis Numbers 19 and 28 where the owner makes payments without delay, but withholds retainage of 10% and 15%, respectively. As a result, the total financing cost jumps from \$42,111 to \$82,550 simply because of the increase in the percentage of retainage.

Table 8. Sensitivity Analysis of the Best and Worst Financing Cases

	Contract terms			Case 1	Case 3
Number	Retained percentage of pay requests (R)	Advance payment percentage (O_{AP})	Lag in paying payment requests (L_P) (months)	Total financing cost of the project (\$)	Total financing cost of the project (\$)
1	0	0	0	14,373	14,753
2	0	0	1	67,354	105,059
3	0	0	2	143,472	232,441
4	0	5	0	0	0
5	0	5	1	40,273	54,699
6	0	5	2	112,269	166,522
7	0	10	0	0	0
8	0	10	1	13,328	14,211
9	0	10	2	81,066	109,365
10	5	0	0	18,348	23,662
11	5	0	1	87,650	149,324
12	5	0	2	167,161	281,807
13	5	5	0	0	0
14	5	5	1	59,769	89,692
15	5	5	2	135,515	214,657
16	5	10	0	0	0
17	5	10	1	31,966	39,747
18	5	10	2	103,869	154,789
19	10	0	0	42,111	70,089
20	10	0	1	120,202	201,038
21	10	0	2	197,695	333,520
22	10	5	0	18,320	18,733
23	10	5	1	92,198	141,406
24	10	5	2	166,083	266,370
25	10	10	0	9,105	9,106
26	10	10	1	64,194	88,972
27	10	10	2	134,471	206,503
28	15	0	0	82,550	121,803
29	15	0	1	154,300	252,752
30	15	0	2	230,872	385,234
31	15	5	0	62,186	69,602
32	15	5	1	126,297	193,119
33	15	5	2	199,263	318,084
34	15	10	0	42,369	42,370
35	15	10	1	98,758	140,685
36	15	10	2	167,685	258,217

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It should also be noted that regardless of changes in the contract terms, the financing cost in Case 3 is larger than in Case 1, and proves that the financing model proposed in this paper always performs better than Case 3 regardless of contract terms. The significance of the proposed model can be more visible when the project duration is longer or the contract terms are unfavorable. For example, in sensitivity analysis Number 30, where the conditions are quite extreme (with 15% retainage, 0% advance payment, and 2 months' delay in owner payments), the difference between Case 1 and Case 3 is $154,362 (i.e., $230,872 subtracted from $385,234), whereas considering the contract terms in Number 20, where the conditions are quite average (with only 10% retainage, no advance payment, and only 1 month delay in owner payments), the difference is only $80,836 (i.e., $120,202 subtracted from $201,038).

Conclusion

The following are the 11 significant improvements that are achieved by using the model proposed in this paper, as compared to previous research.

1. The total financing cost calculated by the proposed model is far less than the financing cost calculated by using the models proposed in earlier research. Contractors benefit from minimizing financing cost.
2. The smaller financing cost generated by the proposed model is largely due to considering different financing alternatives and to performing optimization (never done before in construction projects). The proposed model is flexible enough to accommodate as many financing alternatives as available. A contractor can simply disregard those alternatives that are not available, thereby reflecting the conditions in any real project.
3. In the proposed model, a long-term loan is considered when it avoids a large withdrawal from the line of credit and reduces the interest paid on the contractor's primary account.
4. The credit limit can be either a predetermined or an undetermined amount. The proposed model can handle both situations.
5. Because several financing alternatives are included in the proposed model, contractors do not need to change the start time of activities to satisfy cash constraints. Therefore, financing needs are satisfied without using the total float of activities, which results in reducing the risk of a critical work schedule. However, considering multiple financing alternatives may be time consuming, may necessitate extra effort, and may require services of finance specialists to find and negotiate with appropriate lenders.
6. The work schedule does not need to be extended because of the unavailability of cash caused by unlimited credit limits and/or the availability of different financing alternatives. Because several financing alternatives are provided, the proposed model generates an optimum set of alternatives that allows the contractor to complete the project on schedule in most instances.
7. A financing schedule for both borrowed money and repaid money is provided, which was never done in any of the previous studies.
8. The proposed model allows the contractor to set the minimum cumulative net balance of the cash flow at the end of each period as a contingency.
9. Use of the model can be particularly advantageous when the project duration is long or the contract terms are unfavorable.
10. During the project, an updated financing schedule can be obtained if any of the financing alternatives are changed.
11. Even though the proposed model is used to maximize the profit of the contractor after the contract is signed, it can also be used at the bidding stage to reduce the bidding price and to increase the chance of winning the contract for the contractor.

Given the points mentioned earlier, the proposed model is a distinct improvement over past models, in that it not only minimizes financing cost, but also allows completion on schedule. However, this study has three limitations. First, the proposed model ties direct cost to activity progress, but material expenses can follow a different schedule depending on the procurement plan. Second, considering time-cost tradeoff analysis while optimizing financing cost is not considered in this study. Third, although construction companies may run multiple projects simultaneously, this research proposes a model for a single project. The next stage of this research involves using time-cost tradeoff in order to optimize not only the financing cost but also project duration for maximum profit. In addition, although delaying the start times of activities can result in a more critical schedule, it can reduce the financing cost. This tradeoff needs to be explored in future research.

Data Availability Statement

All data generated or analyzed during the study are included in the published paper. Information about the *Journal*'s data sharing policy can be found here: http://ascelibrary.org/doi/10.1061/%28ASCE%29CO.1943-7862.0001263.

Notation

The following symbols are used in this paper:

AP = advance payment;
B_{ab_t} = amount of borrowed money for short-term loans at the end of period t;
BD = cost of bonding;
$B_{\mathrm{LC}c_t}$ = amount of borrowed money for a line of credit at the end of period t;
B_{LP_t} = amount of borrowed money for long-term loan at the end of period t;
B_t = total borrowed money (financing inflow) at the end of period t;
CBP = contract bid price;
C_{LC_t} = required credit for line of credit at the end of each period t;
CL_{ab} = total credit limit for each financing alternative of loans;
CL'_{ab} = credit limit in each period for short-term loans;
$\mathrm{CL}_{\mathrm{LC}}$ = total credit limit of the line of credit;
$\mathrm{CL}'_{\mathrm{LC}}$ = credit limit in each period for each withdrawal from the line of credit;
$\mathrm{CL}'_{\mathrm{LP}}$ = credit limit for long-term loan;
DC = direct cost of project;
DC_k = net direct cost in week k;
E_t = construction disbursement at the end of each period t;
F_{ab_t} = amount of financing cost for short-term loans at the end of period t;
FC = total financing cost;
$F_{\mathrm{LC}c_t}$ = amount of financing cost for a line of credit at the end of period t;
F_{LP_t} = amount of financing cost for long-term loan at the end of period t;

FOC = fixed overhead cost of project;
F_t = total financing cost (financing outflow) at the end of period t;
L_P = lag in paying payment requests (in months);
L_R = lag to make the final payment and return the retained money (in months);
M = project duration computed in months;
MN = minimum cumulative net balance of the cash flow (including financing flow);
MOB = mobilization cost;
MP = markup of a project;
N'_t = cumulative net balance of the cash flow (including financing flow) in period t;
NF_t = net financing flow at the end of period t;
NFC_{t-1} = cumulative net financing costs from period 0 to period $(t-1)$;
NFC_t = cumulative net balance of the financing cost at the end of period t;
O_{AP} = advance payment percentage of contract bid price;
O_B = bond premium percentage;
O_F = fixed overhead cost per week;
O_M = mobilization cost percentage;
O_{MP} = markup percentage;
O_V = variable overhead percentage of the direct cost;
P_{t+L_P} = progress payment at the end of each period;
P_{t+L_R} = final payment including retainage;
R = retained percentage of pay requests;
R_{ab_t} = amount of repaid money for short-term loans at the end of period t;
R_{LCc_t} = amount of repaid money for a line of credit at the end of period t;
R_{LP_t} = amount of repaid money for long-term loan at the end of period t;
R_t = total repaid money (financing outflow) at the end of period t;
T_{ab} = total amount of borrowed money relative to each short-term loan alternative;
TB_{LC_t} = withdrawal from the line of credit at the end of each period t;
T_F = time of final payment;
$t+L_P$ = time when the progress payment is made;
$t+L_R$ = time when the final payment is made;
TR_t = total money to be repaid including financing cost at the end of period t;
VOC = variable overhead cost of a project; and
W = project duration computed in weeks.

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